CHAPTER 6

RECTIFIERS

Any electrical device which has a high resistance to current in one direction and a low resistance to current in the opposite direction possesses the ability to convert an a-c current into a current which contains a d-c component in addition to a-c components. An ideal rectifier would be one with zero resistance in the forward direction and with an infinite resistance in the reverse direction. A number of devices possess non-linear characteristics, among which are high-vacuum thermionic diodes, gas-filled and vapor-filled thermionic diodes, pool-cathode mercury arcs, and certain crystals.

The important rectifiers for power purposes fall into two general groups, depending on their inherent characteristics. The vacuum tube rectifier possesses an infinite resistance on the inverse cycle, as the tube will not conduct when the plate is negative with respect to the cathode. On the forward, or conducting, portion of the cycle, the vacuum diode is characterized by an almost constant and low value of resistance. The gas or vapor rectifiers also possess an infinite resistance on the inverse cycle, but as discussed in Sec. 1-31, they are characterized by a substantially constant tube drop during conduction. Owing to these differences, the resulting operation in a circuit is slightly different. A detailed discussion is included below.

6-1. Single-phase Half-wave Vacuum Rectifier. The basic circuit for half-wave rectification is shown in Fig. 6-1. It is assumed that the load is a pure resistance. Also, it is supposed that the power transformer is ideal, with negligible resistance and leakage reactance.

An application of Kirchhoff's potential law to the load circuit yields

$$e = e_b + i_b R_l \tag{6-1}$$

where e is the instantaneous value of the applied potential, e_b is the instantaneous potential across the diode when the instantaneous current is i_b, and R_l is the load resistance. This one equation is not sufficient for the determination of the two unknown quantities i_b and e_b that appear in the expression. Here, as for triodes and multielectrode tubes, a second relation is contained in the static plate characteristic of the tube. Consequently a solution is effected by drawing the load line on the plate characteristic.

There is one significant difference between the solution of the diode as a rectifier and that for the other tubes as amplifiers. With the rectifier, an a-c source supplies the power to the circuit. A vacuum tube as an amplifier converts direct current from the plate supply into alternating current.

The dynamic characteristic of the rectifier circuit is obtained in a somewhat different manner from the corresponding curve for an amplifier.

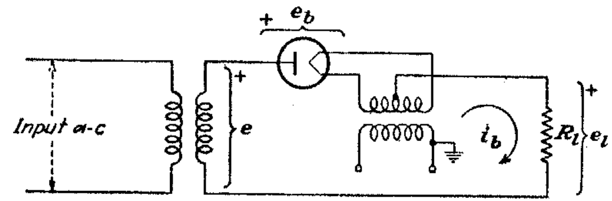


Fig. 6-1. A simple half-wave rectifier circuit.

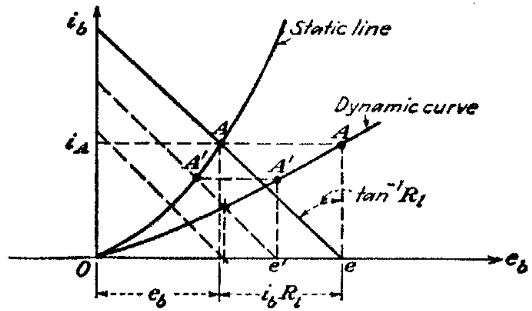


Fig. 6-2. The static and dynamic characteristics of a rectifier.

The procedure is illustrated in Fig. 6-2. For an applied potential e, the current is the intersection of the load line with the static characteristic, say point A. That is, for the particular circuit, the application of the potential e results in a current i_A. This is one point on the dynamic curve and is drawn vertically above e in the diagram. The slope of the load line does not vary, although the intersection with the e_b axis varies with e. Thus, when the applied potential has the value e', the corresponding current is i_A'. The resulting curve so generated is the dynamic characteristic.

If the static characteristic of the tube were linear, the dynamic characteristic would also be linear. Note from the construction, however, that there is considerably less curvature in the dynamic curve than there is in the static characteristic. It will be assumed in what follows that the dynamic curve is linear.

To find the waveshape of the current in the output circuit, the procedure followed is that illustrated in Fig. 6-3. This procedure is very much like that used to find the waveshape in a general amplifier circuit; in fact, the situation here is quite like that of a class B amplifier, except that cutoff of the tube exists at zero input.

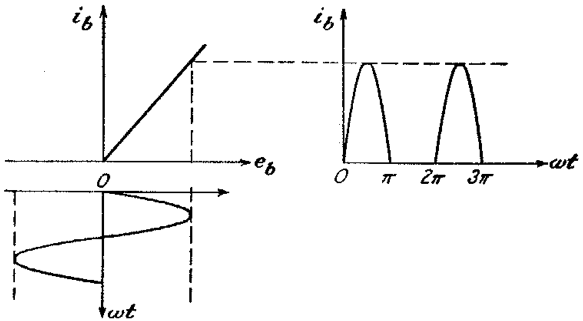


FIG. 6-3. The method of obtaining the output-current waveform from the dynamic characteristic.

If it is assumed that the relation

$$e_b = i_b r_p \tag{6-2}$$

is valid during conduction, and this supposes that the static characteristic is linear, then from Eq. (6-1) it follows that

$$e = e_b + i_b R_l = i_b(r_p + R_l) = E_m \sin \omega t \tag{6-3}$$

or

$$\begin{aligned} i_b &= \frac{E_m}{R_l + r_p} \sin \omega t = I_m \sin \omega t \qquad &\text{when } 0 \leqq \omega t \leqq \pi \\ i_b &= 0 &\text{when } \pi \leqq \omega t \leqq 2\pi \end{aligned} \tag{6-4}$$

where

$$I_m = \frac{E_m}{R_l + r_p}$$

The d-c power supplied to the load is defined as the product of the reading of a d-c ammeter in the load circuit and a d-c voltmeter across the load. Thus

$$P_{\text{d-c}} = E_{\text{d-c}} I_{\text{d-c}} \tag{6-5}$$

Clearly, the reading of the d-c ammeter is represented by

$$I_{\text{d-c}} = \frac{1}{2\pi} \int_0^{2\pi} i_b \, d\alpha = \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha \, d\alpha = \frac{I_m}{\pi} \tag{6-6}$$

and so

$$P_{\text{d-c}} = I_{\text{d-c}}^2 R_l = \left(\frac{1}{\pi}\right)^2 \frac{E_m^2 R_l}{(r_p + R_l)^2} \tag{6-7}$$

The power supplied to the circuit from the a-c source, and this is the power that would be read by a wattmeter with its current coil in the line and with the potential coil across the source, is given by the integral

$$P_i = \frac{1}{2\pi} \int_0^{2\pi} e i_b \, d\alpha \tag{6-8}$$

This becomes, by Eqs. (6-2) and (6-3),

$$P_i = \frac{1}{2\pi} \int_0^{2\pi} i_b^2 (r_p + R_l) \, d\alpha \tag{6-9}$$

which may be written in the form

$$P_i = I_{\text{rms}}^2 (r_p + R_l) \tag{6-10}$$

where the rms current has the value

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_b^2 \, d\alpha} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \alpha \, d\alpha} = \frac{I_m}{2} \tag{6-11}$$

The efficiency of rectification is defined by the relation

$$\eta_r = \frac{P_{\text{d-c}}}{P_i} \times 100\% = \frac{I_{\text{d-c}}^2 R_l}{I_{\text{rms}}^2 (r_p + R_l)} \times 100\%$$

which becomes

$$\eta_r = \left(\frac{I_{\text{d-c}}}{I_{\text{rms}}}\right)^2 \frac{100}{1 + r_p/R_l} \tag{6-12}$$

By combining this with Eqs. (6-6) and (6-11), there results

$$\eta_r = \left(\frac{I_m/\pi}{I_m/2}\right)^2 \frac{100}{1 + r_p/R_l} = \frac{40.6}{1 + r_p/R_l} \quad \% \tag{6-13}$$

This indicates that the theoretical maximum efficiency of the single-phase half-wave rectifier is 40.6 per cent. But it may be shown that maximum power output occurs when $R_l = r_p$, with a corresponding theoretical plate-circuit efficiency of 20.3 per cent.

There are several features of such a rectifier circuit that warrant special attention. Refer to Fig. 6-1, which shows the complete wiring diagram of the rectifier. On the inverse cycle, i.e., on that part of the cycle during which the tube is not conducting, the maximum potential across the rectifier tube is equal to the transformer maximum value. That is, the peak inverse potential across the tube is equal to the transformer maximum value.

Note also from the diagram that with the negative terminal of the output connected to ground the full transformer potential exists between the primary and the secondary windings of the filament heating transformer. This requires that the transformer insulation must be adequate to withstand this potential without rupture. Evidently if the positive terminal is grounded, then the transformer need not have a high insulation strength.

6-2. Ripple Factor. Although it is the object of a rectifier to convert a-c into d-c current, the simple circuit considered does not achieve this. Nor, in fact, do any of the more complicated rectifier circuits, either single-phase or polyphase, accomplish this exactly. What is achieved is a unidirectional current, periodically fluctuating components still remaining in the output. Filters are ordinarily used in rectifier systems in order to help decrease these fluctuating components, and these will receive detailed consideration in Chap. 7. A measure of the fluctuating components is given by the ripple factor r, which is defined as

$$r = \frac{\text{rms value of a-c components of wave}}{\text{avg or d-c value of wave}}$$

and which may be written as

$$r = \frac{I'_{\text{rms}}}{I_{\text{d-c}}} = \frac{E'_{\text{rms}}}{E_{\text{d-c}}} \tag{6-14}$$

where I'_{rms} and E'_{rms} denote the rms values of the a-c components only.

An analytical expression for the ripple factor is readily possible. It is noted that the instantaneous a-c component of the current is given by

$$i' = i - I_{\text{d-c}}$$

But by definition

$$I'_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (i - I_{\text{d-c}})^2\, d\alpha} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (i^2 - 2iI_{\text{d-c}} + I_{\text{d-c}}^2)\, d\alpha}$$

This expression is readily interpreted. The first term of the integrand when evaluated yields the square of the rms value of the total wave I_{rms}^2. The second term yields

$$\frac{1}{2\pi}\int_0^{2\pi} 2iI_{\text{d-c}}\, d\alpha = 2I_{\text{d-c}}^2$$

The rms ripple current then becomes

$$I'_{\text{rms}} = \sqrt{I_{\text{rms}}^2 - 2I_{\text{d-c}}^2 + I_{\text{d-c}}^2} = \sqrt{I_{\text{rms}}^2 - I_{\text{d-c}}^2}$$

By combining these results with Eq. (6-13)

$$r = \frac{\sqrt{I_{\text{rms}}^2 - I_{\text{d-c}}^2}}{I_{\text{d-c}}} = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{d-c}}}\right)^2 - 1} \tag{6-15}$$

This expression is independent of the current waveshape and applies in general, since the development was not confined to a particular waveshape. In the case of the half-wave single-phase rectifier the ratio

$$\frac{I_{\text{rms}}}{I_{\text{d-c}}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$$

and hence

$$r = \sqrt{1.57^2 - 1} = 1.21 \tag{6-16}$$

This shows that the rms value of the ripple potential exceeds the d-c potential of the output. This merely tends to show that a single-phase half-wave rectifier without filter is a relatively poor device for converting a-c into d-c potential.

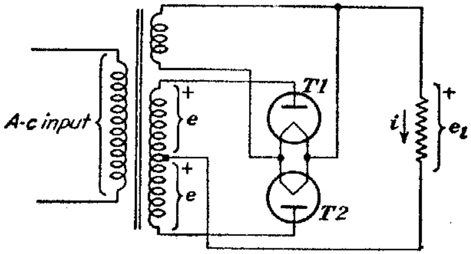


FIG. 6-4. Schematic wiring diagram of a single-phase full-wave rectifier.

6-3. Single-phase Full-wave Rectifier. The circuit of the single-phase full-wave rectifier is given in Fig. 6-4. Actually the circuit comprises two half-wave circuits which are so connected that conduction takes place through one tube during one half of the total power cycle and through the other tube during the second half of the power cycle. The output current through the load has the form illustrated in Fig. 6-5, where the portions of the wave marked 1 flow through tube $T1$ and the portions of the wave marked 2 flow through tube $T2$.

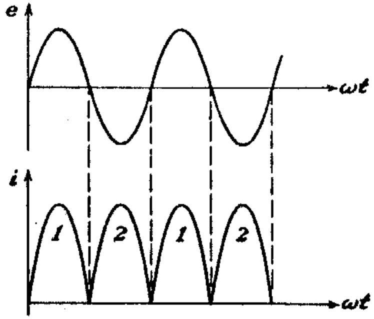


FIG. 6-5. The transformer potential and output load current in a single-phase full-wave rectifier.

The d-c and rms values of the load current are found from Eqs. (6-6) and (6-11) to be

$$I_{\text{d-c}} = \frac{2I_m}{\pi}$$
$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \tag{6-17}$$

where I_m is the peak value of the current wave. The d-c output power is then

$$P_{\text{d-c}} = I_{\text{d-c}}^2 R_l = \left(\frac{2}{\pi}\right)^2 \frac{E_m^2 R_l}{(r_p + R_l)^2} \tag{6-18}$$

By comparing this expression with Eq. (6-7) it is seen that the power delivered to the load is higher by a factor of 4 in the full-wave case. However, the power depends on the circuit parameters in the same way as for the half-wave circuit.

The input power from the a-c source is readily found to have the same form as Eq. (6-10), viz.,

$$P_i = I_{\text{rms}}^2(r_p + R_l) \tag{6-19}$$

The efficiency of rectification is

$$\eta_r = \frac{81.2}{1 + r_p/R_l} \quad \% \tag{6-20}$$

This expressions shows a theoretical maximum that is twice that of the half-wave rectifier.

The ripple factor is readily found when it is noted that

$$\frac{I_{\text{rms}}}{I_{\text{d-c}}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

From Eq. (6-15),

$$r = \sqrt{1.11^2 - 1} = 0.482 \tag{6-21}$$

Thus the ripple factor has dropped from 1.21 in the half-wave rectifier to 0.482 in the present case. What has been accomplished in the full-wave rectifier, therefore, is that the rectification process has become more efficient, with a higher percentage of the power supplied to the circuit being converted into the desired d-c power, and with a consequent smaller fraction remaining in a-c form, which, while producing heating of the load, does not contribute to the desired d-c power.

A study of Fig. 6-4 indicates that when one tube is conducting, say $T1$, then tube $T2$ is in the nonconducting state. Except for the tube drop $i_b r_p$ in $T1$, the peak inverse potential across $T2$ is $2E_m$, or twice the transformer maximum potential measured to the mid-point, or the full transformer potential. The potential stress between windings of the filament transformer is seen to be the full d-c potential, if the negative is grounded, and is sensibly zero, if the positive is grounded.

6-4. Circuits with Gas Diodes. Gas diodes may be used in the half-wave and full-wave circuits discussed above. Owing to their different plate characteristics, the results are somewhat different. For these tubes a sensibly constant potential appears across the tube when the tube is conducting, but conduction does not begin until the applied potential exceeds the breakdown potential of the tube. The tube will consequently conduct for less than 180 deg in each cycle. The situation is illustrated in Fig. 6-6.

The equation of the potential across the load during conduction is obtained by applying Kirchhoff's law to the plate circuit,

$$e_l = i_b R_l = E_m \sin\alpha - E_0 \tag{6-22}$$

and the corresponding expression for the current is

$$i_b = \frac{E_m \sin\alpha - E_0}{R_l} \tag{6-23}$$

where E_0 is the constant tube drop during conduction.

The d-c plate current is found by taking the average value of the instantaneous current and is

$$I_{\text{d-c}} = \frac{1}{2\pi}\int_{\alpha_1}^{\alpha_2} \frac{E_m \sin\alpha - E_0}{R_l}\, d\alpha \tag{6-24}$$

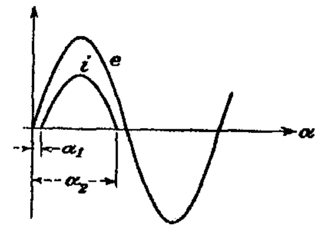


Fig. 6-6. The applied potential and the current waveshape in a half-wave rectifier circuit using a gas diode.

where α_1 is the angle at which the tube fires and α_2 is the angle at which conduction ceases. Ordinarily the applied plate potential is much larger than E_0, and the angles α_1 and $\pi - \alpha_2$ are very nearly zero. Consequently the limits on the integral of Eq. (6-24) may be changed to 0 and π without appreciable error in the result. When this is done and the integral is evaluated, it is found that

$$I_{\text{d-c}} = \frac{E_m}{\pi R_l} - \frac{E_0}{2R_l} = \frac{E_m}{\pi R_l}\left(1 - \frac{\pi}{2}\frac{E_0}{E_m}\right) \tag{6-25}$$

The load potential $E_{\text{d-c}}$ may be written as

$$E_{\text{d-c}} = \frac{E_m}{\pi}\left(1 - \frac{\pi}{2}\frac{E_0}{E_m}\right) \tag{6-26}$$

This equation does not contain the load current. This means, of course, that $E_{\text{d-c}}$ is independent of the load current, with consequent perfect regulation.

To calculate the efficiency of rectification, it is necessary to calculate the input power to the plate circuit. This is given by

$$P_i = \frac{1}{2\pi}\int_0^{\pi} e i_b\, d\alpha = \frac{1}{2\pi}\int_0^{\pi} E_m \sin\alpha\, \frac{E_m \sin\alpha - E_0}{R_l}\, d\alpha$$

where the limits are again taken as 0 and π. This expression reduces to

$$P_i = \frac{E_m^2}{4R_l}\left(1 - \frac{4}{\pi}\frac{E_0}{E_m}\right) \tag{6-27}$$

The efficiency of rectification is then

$$\eta_r = \frac{P_{d\text{-}c}}{P_i} = \frac{4}{\pi^2} \frac{\left(1 - \frac{\pi}{2}\frac{E_0}{E_m}\right)^2}{1 - \frac{4}{\pi}\frac{E_0}{E_m}} \tag{6-28}$$

which may be reduced to the form

$$\eta_r = 40.6\left(1 - 1.87\frac{E_0}{E_m}\right) \quad \% \tag{6-29}$$

Note that this value is independent of the load current or load resistance. To the same approximation, namely, $E_m \gg E_0$, the ripple factor is given by

$$r = 1.21\left(1 + 0.5\frac{E_0}{E_m}\right) \tag{6-30}$$

which is slightly higher than the value with the vacuum diode. This increased ripple results because the tube conduction is less than 180 deg.

The corresponding properties of the full-wave circuit with gas tubes will follow a completely parallel development and yield results that bear the same relation to the vacuum-tube case that the foregoing results do to the corresponding half-wave vacuum-rectifier case.

6-5. Miscellaneous Single-phase Rectifier Circuits. A variety of other rectifier circuits exist which find widespread use. Among these are bridge rectifier circuits, potential-doubling circuits and potential-multiplying circuits. The bridge circuit finds extensive use both as a power rectifier and also as the rectifying system in rectifier-type a-c meters. The rectifiers for power use utilize thermionic diodes of both the vacuum and gas varieties, whereas those for instrument use are usually of the copper oxide or crystal types.

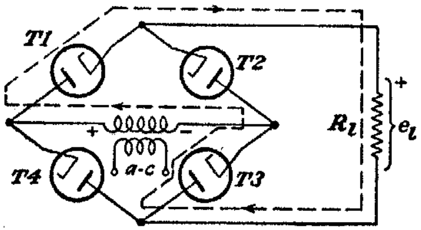


FIG. 6-7. Single-phase full-wave bridge rectifier circuit.

To examine the operation of the bridge circuit, refer to Fig. 6-7. It is observed that two tubes conduct simultaneously during one half of the cycle and the other two tubes conduct during the second half of the cycle. The conduction paths and directions are such that the resulting current through the load is substantially that shown in Fig. 6-5.

The primary features of the bridge circuit are the following: The currents drawn in both the primary and secondary of the plate-supply transformer are sinusoidal. This permits a smaller transformer to be used for a given output power than is necessary for the same power with the single-phase full-wave circuit of the two-tube type.* Also, the transformer need not have a center tap. Since each tube has only transformer potential across it on the inverse cycle, the bridge circuit is suitable for high-potential applications. However, the transformers supplying the heaters of the tubes must be properly insulated for the high potential.

A rectifier meter is essentially a bridge-rectifier system which utilizes copper oxide elements. The potential to be measured is applied through a multiplier resistance to two corners of the bridge, a d-c milliammeter being used as an indicating instrument across the other two corners. But as the d-c milliammeter reads average values of current, the scale of the meter is calibrated to give rms values of sinusoidal waves by applying a sinusoidal potential to the input terminals. The indication on such an instrument is not correct for input signals that contain appreciable harmonics.

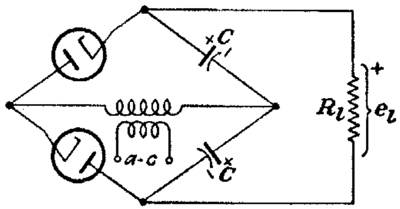


FIG. 6-8. A full-wave potential-doubling circuit.

A common potential-doubling circuit is shown in Fig. 6-8. The output[1] from such a circuit is approximately equal to twice the transformer maximum potential. It operates by alternately charging each of the two capacitors to the transformer peak potential E_m, current being continually drained from the capacitors through the load. This circuit is characterized by poor regulation unless very large capacitors are used. The peak inverse potential is twice the transformer peak potential. If ordinary rectifiers are used, two separate filament sources are required. If a relatively low potential system is built, and these are used extensively in a-c/d-c radio sets, special tubes such as 25Z5 are available. These tubes are provided with separate indirectly heated cathodes. The cathodes in these tubes are well insulated from the heaters, which are connected in series internally.

The regulation of the potential doubler can be improved, particularly at the higher loads, by employing a bridge double rectifier,[2] which is illustrated in Fig. 6-9. The feature of this rectifier circuit is that at light loads the output potential is approximately twice the transformer peak potential. However, the potential will never fall below the output of the bridge circuit at any load, nor will the ripple factor exceed that of the bridge circuit, viz., $r = 0.482$. Most other features of this circuit are like those in the normal bridge circuit, such as the peak inverse potential to which each tube is subjected, and the heater-cathode insulation problems.

An alternative potential-doubling circuit[3] is shown in Fig. 6-10. The output potential from this circuit, like that from Fig. 6-8, is approximately equal to twice the transformer maximum potential. It operates

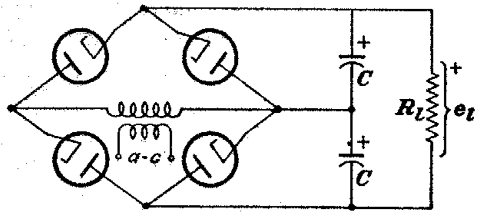


FIG. 6-9. A bridge doubler circuit.

by charging capacitor C_1 during one half cycle through tube $T1$ to the transformer peak potential E_m and during the next half cycle charges C_2 through tube $T2$ to the potential determined by that across C_1 and the transformer in series, the peak being approximately $2E_m$. The peak inverse potential across each tube is twice the transformer peak potential.

This circuit may be extended to a quadrupler by adding two tubes and two capacitors, as shown in Fig. 6-11. It may be extended to provide n-fold multiplication, odd or even.

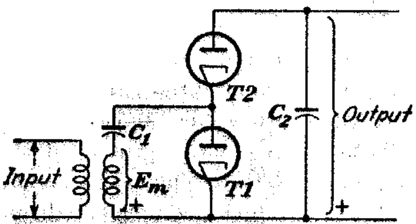


FIG. 6-10. A half-wave potential-doubling circuit.

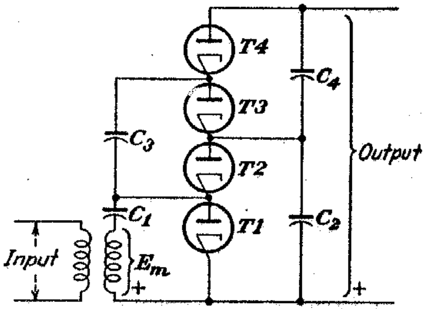


FIG. 6-11. A half-wave potential-quadrupling circuit.

6-6. Controlled Rectifiers. It is sometimes necessary to provide a control on the amount of the rectified current in a rectifier. Such controlled currents are required in motor speed control, in certain electric welding operations, in lighting-control installations in theaters, in torque controls of various types, and in a variety of industrial control applications. Such control of the output of a rectifier may be accomplished by controlling the potential of the power transformer feeding the rectifier, or this might, in some cases, be controlled by inserting a controlling resistor in the output circuit. The first method may require expensive control equipment, and the second would be characterized by poor efficiency. The development of thyratrons, ignitrons, and excitrons, the latter two being single-anode pool tanks, has made control a direct and relatively inexpensive process. The basic analyses are common to the three types of tubes, and no loss of generality is incurred by confining our attention to thyratrons.

In the discussion in Sec. 1-33 it was indicated that the complete electrostatic shielding of the cathode by the massive grid structure in the thyratron (and also in the excitron) provides a means for controlling the initiation of the arc in the tube by controlling the grid potential. Thus

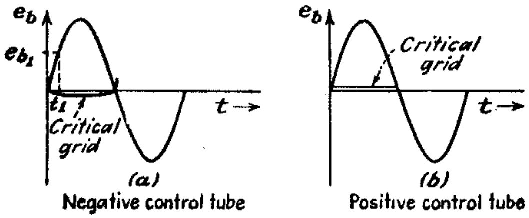


FIG. 6-12. The grid-control curve for an applied sinusoidal plate potential to a thyratron.

since the arc is extinguished once each cycle on each negative half cycle, provided that the arc is initiated regularly, control is possible if the point in each half cycle at which the arc is initiated can be controlled. Such regular control is possible, thus providing a control on the average rectified current.

In order to analyze the action of a thyratron in a controlled circuit, use is made of the critical grid breakdown characteristic of Fig. 1-44. As indicated in Sec. 1-33, a knowledge of this single curve is sufficient to predict the behavior of the thyratron in the control circuit. This curve gives the minimum grid potential required for conduction to occur for each value of plate potential. Thus if a sinusoidal plate potential is applied to the tube, the potential on the grid that is required just to permit conduction at each point in the cycle is found from the critical grid curve. The conditions are illustrated in Fig. 6-12. On this diagram, corresponding to any time t_1 in the positive half cycle, the plate potential is e_{b1}. The corresponding critical grid potential is obtained directly from the critical grid breakdown characteristic and is shown in the figure. This means that, unless the grid potential is more positive than that given by e_{cg} at the particular instant, conduction will not take place. Of course, once conduction begins at any point in the cycle, the grid loses control of the arc and cannot again control until the arc is extinguished.

Suppose that the circuit is so arranged that the grid potential exceeds the critical grid breakdown value at some angle, say φ. Conduction will start at this point in the cycle. But the potential drop across the tube during conduction of the thyratron, like that in any gas arc discharge, remains substantially constant at a low value E_0 that is independent of the current. Consequently the current in the plate circuit of the tube is readily found. Refer to Fig. 6-13, which gives the schematic diagram of a thyratron circuit, and to Fig. 6-14, which shows the potential and current waveshapes in the thyratron.

If the tube drop after conduction has begun is E_0, then the current in the plate load of resistance R_l during the conducting portion of the cycle

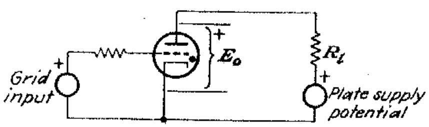


FIG. 6-13. A thyratron circuit with a-c plate and grid excitation.

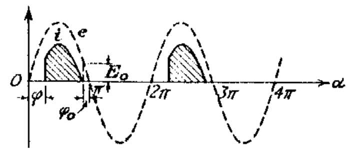


FIG. 6-14. The plate potential and the plate current in a thyratron.

is

$$i_b = \frac{E_m \sin \alpha - E_0}{R_l} \tag{6-31}$$

where E_m is the maximum value of the applied potential. Clearly, the current is zero until conduction takes place, after which it assumes the value dictated by Eq. (6-31). The current follows the form of Eq. (6-31) until the supply potential e falls below E_0 at the phase $\pi - \varphi_0$. The current remains zero until the phase φ is again reached in the next cycle.

The average rectified current, i.e., the value read on a d-c ammeter, is given by the expression

$$I_{\text{d-c}} = \frac{1}{2\pi}\int_{\varphi}^{\pi-\varphi_0} i_b \, d\alpha$$
$$= \frac{E_m}{2\pi R_l}\int_{\varphi}^{\pi-\varphi_0}\left(\sin\alpha - \frac{E_0}{E_m}\right)d\alpha$$

which integrates to

$$I_{\text{d-c}} = \frac{E_m}{2\pi R_l}\left[\cos\varphi + \cos\varphi_0 - \frac{E_0}{E_m}(\pi - \varphi_0 - \varphi)\right] \tag{6-32}$$

where $\alpha = \omega t$, and where φ_0 is defined by the relation

$$E_0 = E_m \sin \varphi_0 \tag{6-33}$$

If the ratio E_0/E_m is very small, then φ_0 may be taken as zero. Equation (6-32) reduces under this condition to the form

$$I_{\text{d-c}} \doteq \frac{E_m}{2\pi R_l}(1 + \cos\varphi) \tag{6-34}$$

The limits of variation of the angle are from 0 to π.

It is clear from this analysis that the average rectified current can be controlled by varying the position in the cycle at which the grid potential exceeds the critical grid starting value. The maximum current is obtained when the arc is initiated at the beginning of each cycle. The minimum current is obtained when the arc is not initiated, and this would occur if initiation were adjusted to occur at the end of each cycle. Sketches showing the character of the results are given in Fig. 6-15.

The potential across the thyratron throughout the cycle will vary in the manner illustrated in Fig. 6-16. During the portion of the cycle when the tube is not conducting current, the full applied potential appears across the tube. During the portion of the cycle when the tube is conducting, the drop across the tube is E_0, which is assumed constant and independent of the tube current.

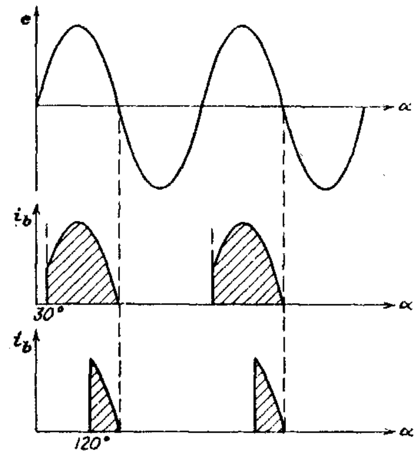


FIG. 6-15. The character of the conduction in a thyratron for various angles of initiation of the arc.

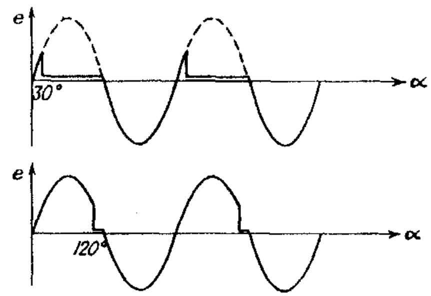


FIG. 6-16. The potential across the thyratron for various current-conduction periods.

The reading of a d-c voltmeter placed across the tube will be

$$E_{\text{d-c}} = \frac{1}{2\pi}\int_0^{2\pi} e_b \, d\alpha$$

This integral is written as

$$E_{\text{d-c}} = \frac{1}{2\pi}\left(\int_0^{\varphi} E_m \sin\alpha \, d\alpha + \int_{\varphi}^{\pi-\varphi_0} E_0 \, d\alpha + \int_{\pi-\varphi_0}^{2\pi} E_m \sin\alpha \, d\alpha\right)$$

This integrates to

$$E_{\text{d-c}} = \frac{E_0}{2\pi}(\pi - \varphi_0 - \varphi) - \frac{E_m}{2\pi}(\cos\varphi + \cos\varphi_0) \tag{6-35}$$

If the peak transformer potential $E_m \gg E_0$, this equation reduces to

$$E_{\text{d-c}} \doteq -\frac{E_m}{2\pi}(1 + \cos\varphi) \tag{6-36}$$

The appearance of the negative sign merely means that the cathode is more positive than the plate for most of the cycle. It is to be emphasized that the d-c voltmeter does not read the value E_0 when an a-c potential is applied to the tube but will read E_0 if a d-c potential is applied.

The readings of d-c and a-c indicating instruments may be calculated in somewhat similar ways. For example, the reading of an a-c ammeter in the plate lead will be

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_b^2 \, d\alpha}$$

which is

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\varphi}^{\pi-\varphi_0} \left(\frac{E_m \sin\alpha - E_0}{R_l}\right)^2 d\alpha} \qquad (6\text{-}37)$$

This integration is readily effected. Similarly, the reading of a wattmeter to indicate the total power supplied to the plate circuit is

$$P = \frac{1}{2\pi}\int_0^{2\pi} e i_b \, d\alpha$$

which is

$$P = \frac{1}{2\pi}\int_{\varphi}^{\pi-\varphi_0} E_m \sin\alpha \, \frac{E_m \sin\alpha - E_0}{R_l} \, d\alpha \qquad (6\text{-}38)$$

This integration is also readily effected. In particular, it is essential to set up the basic expression for any quantity before proceeding in its evaluation.

6-7. Phase-shift Control. In the phase-shift method of control, the conduction point of the cycle is controlled by controlling the phase angle

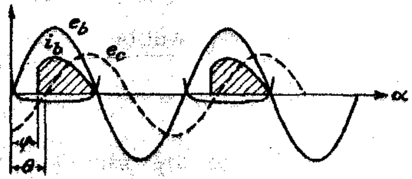


FIG. 6-17. Phase-shift control of a thyratron.

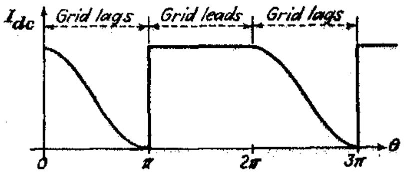


FIG. 6-18. The average load current as a function of phase angle between the grid and plate potentials.

between the plate and grid potentials. The situation is illustrated in Fig. 6-17. In this figure, the grid-cathode potential e_c lags the plate-cathode potential e_b by the angle θ. Note from the figure that the grid potential equals the critical grid breakdown potential at the angle φ and conduction begins at this point in the cycle. The arc is extinguished when the plate potential falls below the value to maintain conduction.

If the applied grid potential is large compared with the critical grid potential at the point of conduction, the angles φ and θ are approximately the same. Also, it may be assumed under these conditions that the critical grid curve coincides with the zero-potential axis. Also, if $E_m \gg E_0$, then Eq. (6-34) will give the dependence of d-c load current on the phase angle for all values of φ for which the grid potential lags behind the plate potential. When the grid potential leads the plate potential by any angle, conduction will occur very nearly at the beginning of each cycle, with full rectification. A sketch showing these results is given in Fig. 6-18. The curve possesses a discontinuity at the 180-deg position, since for an angle slightly larger than 180 deg the plate current

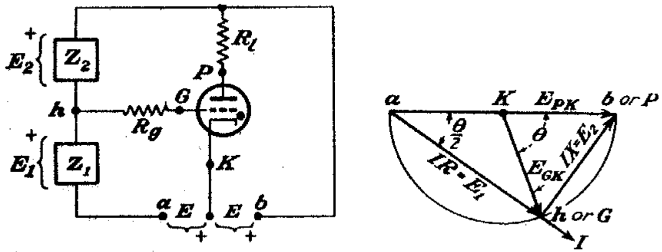


FIG. 6-19. A simple phase-shifting network and the potential circle diagram.

is at its full value, whereas for an angle slightly less than 180 deg the plate current is zero.

For those cases where a small plate potential or a small grid potential is used, the foregoing simplifications are not valid. However, the analysis follows the same general form, due account being taken of the difference between φ and θ and of the angles $\pi - \varphi_0$ and π.

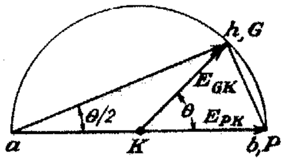
Circuits for achieving the phase-shift control in a single-phase system are readily analyzed. A common circuit and the potential sinor diagram that applies to this circuit are given in Fig. 6-19. The phase between the grid and the plate potentials is controlled by means of the two impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$. It should be noted that the potential circle diagram in Fig. 6-19 has been drawn under the assumption that

$$\mathbf{Z}_1 = R \text{ (a resistance)} \qquad \mathbf{Z}_2 = j\omega L \text{ (an inductance)}$$

Also, the potential circle diagram applies only during the periods before conduction begins in each cycle. Before conduction begins, there is no current in the load R_l, and points b and P are the same. From the results of the potential circle diagram, it is clear that the phase θ between the plate and grid potentials can be varied over the range from 0 to 180 deg by varying the control resistance R in the phase-shifting network, with θ at 180 deg when $R = 0$ and with θ at 0 deg when $R = \infty$. Evidently, the load current will decrease as the resistance R decreases. The phase angle θ is, from inspection of Fig. 6-19,

$$\tan\frac{\theta}{2} = \frac{E_2}{E_1} = \frac{Z_2}{Z_1} \qquad (6\text{-}39)$$

If the control impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are interchanged, then the sinor diagram that results has the form given in the accompanying diagram,



which shows that E_{gk} leads the potential E_{pk}. Then, from Fig. 6-18, no control over the plate current is possible, and $I_{d\text{-}c}$ is a constant and equal to its maximum value for all values of θ. The use of R and C as control impedances is possible and is generally preferred over the use of R and L. With an RC phase-shifting circuit, control is possible for $\mathbf{Z}_1 = -jX_C$, $\mathbf{Z}_2 = R$ but is not possible for $\mathbf{Z}_1 = R$, $\mathbf{Z}_2 = -jX_C$.

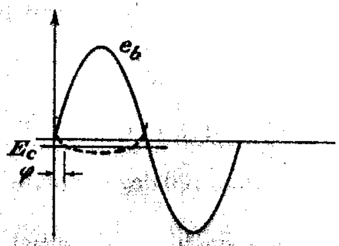


FIG. 6-20. Direct-current bias control of a thyratron.

6-8. D-C Bias Control. The magnitude of the d-c or average rectified current of a thyratron may be controlled by applying a d-c bias to the grid of the tube and controlling its magnitude. The plate supply must be an a-c potential. The situation is best understood by reference to Fig. 6-20.

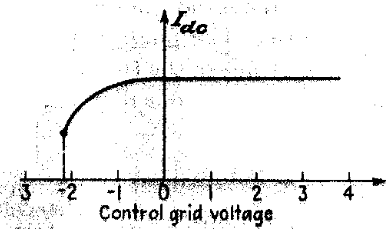


FIG. 6-21. The average plate current in a thyratron as a function of the d-c grid-bias potential.

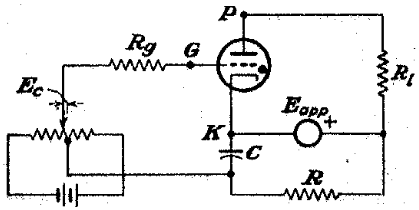


FIG. 6-22. A circuit for bias phase control of a thyratron.

The tube will conduct at the point where E_0 intersects the critical grid curve, the angle φ in the diagram. Clearly, if the negative grid potential is too large to intersect the critical grid curve, no conduction will be possible. The optimum bias is that for which the grid-potential line is tangent to the critical grid curve, and the tube conducts for one-half of the cycle. For less negative bias, the conduction angle φ is less than 90 deg. Control is evidently possible over the range from full conduction to half conduction. The results of Fig. 6-21 show the character of the control.

6-9. Bias Phase Control. The combination of d-c and a-c potentials as a bias yields bias phase control. A circuit for such control is given in Fig. 6-22. The network comprising R and C serves to introduce an a-c potential of fixed phase with respect to the plate potential.

Suppose, for convenience, that $R = X_C$. The a-c grid potential will then lag the plate potential by 45 deg. The amplitude of the a-c grid potential is 0.707 that of the plate potential. Suppose also that the critical grid potential coincides with the zero axis. The general features of the operation of this control are illustrated in Fig. 6-23. The conditions corresponding to three different values of E_C are illustrated. In the first, E_C is positive; in the second, E_C is zero; in the third, E_C is negative.

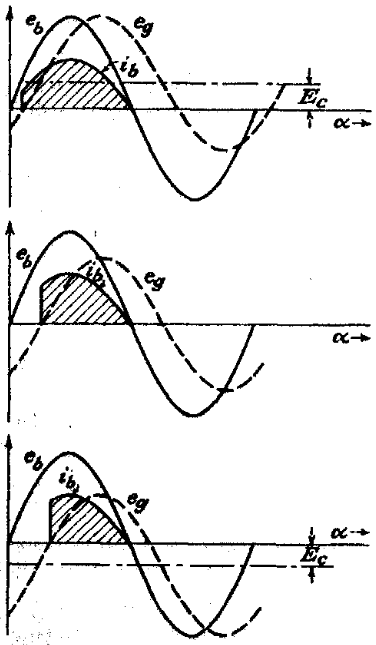


FIG. 6-23. Bias phase control, showing current variations for three different values of d-c component E_C.

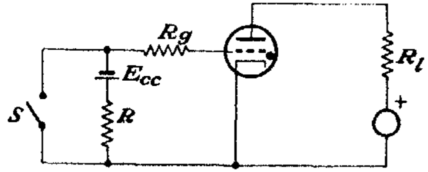


FIG. 6-24. An on-off thyratron control circuit.

For the circuit shown, the minimum rectified current occurs when E_C is negative and equal to E_{gm}. Conduction begins at 135 deg in the cycle and continues until the end of the cycle. If the d-c bias is made more negative than this, no conduction is possible. It is evident from the diagrams that conduction will begin at the start of the cycle when the d-c bias equals 0.707 times E_{gm}.

6-10. On-Off Control. A variety of circuits exist which permit on-off control. Such circuits would be used when it is desired to use a thyratron as an arcless switch or contactor. A circuit for on-off control is given in Fig. 6-24. With switch S open, no conduction occurs since E_{cc} is so adjusted that it is more negative than the maximum negative critical grid value. When the switch S is closed, the grid is tied to the cathode and approximately maximum rectified current is delivered. The resistance R serves to prevent short circuiting of the battery source E_{cc} when S is closed.

6-11. Control of Ignitrons. As discussed in Sec. 1-35, the ignitron will not conduct at any portion of the cycle of an applied a-c potential to the plate unless ignition is caused by applying a potential to the igniter rod. Moreover, since this ignition pulse may be applied at any point in the cycle, control of the average current is afforded by controlling the ignition of the tube. However, the instantaneous power required by the igniter circuit of the ignitron is higher than that required by the grid circuit of a thyratron, and the methods of thyratron control are not applicable to ignitrons. But with control accomplished the general discussion of Sec. 6-6 is applicable, and Eqs. (6-31) to (6-38) express the results for the ignitron as well as for the thyratron.

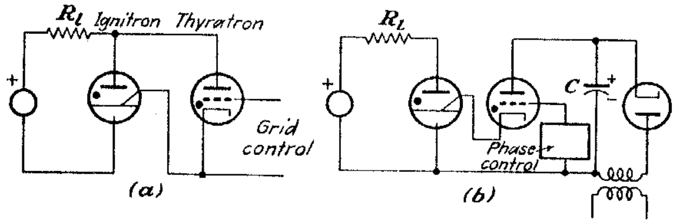


FIG. 6-25. Several ignitron control circuits.

Several common methods of establishing the point of ignition in each cycle of an ignitron are illustrated in Fig. 6-25. These methods utilize thyratrons for the control of the ignitron. In the first of these diagrams, the ignitron current passes through the load resistor, whereas, in the second, the igniter-rod current does not pass through the load.

Refer to Fig. 6-25*b*, and suppose that the thyratron is not conducting. In this circuit the capacitor C will be charged to the peak value of the transformer potential through the rectifier. If the thyratron grid potential is adjusted to permit conduction, the capacitor charge will pass through the thyratron and igniter-rod circuits, and the ignitron will conduct provided that the ignitron anode potential is sufficiently positive to maintain the discharge. The current surge through the ignitron-rod circuit will quickly discharge the capacitor, the thyratron anode potential will fall below that needed to maintain the arc, and the thyratron igniter-rod circuit current will fall to zero. The capacitor will be recharged through the diode rectifier circuit in time to control the ignition point in the next cycle.