ELECTRON TUBES

ELECTRON EMISSION

All electron tubes depend for their operation on the flow of electrons within the tube, either through high vacuum or an ionized gas. The electrons are emitted from a cathode surface as a result of one of four processes that are distinguished on the basis of the mechanism by which the electrons are enabled to leave the surface. These processes are elevated temperature (thermonic or primary emission); bombardment by other particles, generally electrons (secondary emission); the action of a high electric field (field emission); or the incidence of photons (photoemission).

Thermionic Emission

Thermionic emission occurs when the electrons in the cathode material have enough thermal energy to overcome the forces at the surface and escape.

The thermal emission of electrons from metals obeys the Richardson–Dushman equation

$$J_0 = A T^2 \exp\left(-\frac{\phi_e}{kT}\right)$$

where $J_0$ is emission density in amperes/cm², $A$ is a constant [amperes/cm²(K)²], $\phi_e$ is the work function [electron-volts], and $T$ is temperature in degrees Kelvin. $A$ and $\phi_e$ are characteristic of the specific material.

The current density given by this equation is usually referred to as the saturation emission current density. Typical constants are given in Table 1 for several commonly used cathode materials.

The maximum current of which a cathode is capable at the operating temperature is known as the saturation current and is normally taken as the value at which the current first fails to increase as the three-halves power of the voltage causing the current to flow. Thoriated-tungsten filaments for continuous-wave operation are usually assigned an available emission of approximately half the saturation value. Oxide-coated emitters do not have a well-defined saturation point and are designed empirically. The available emission from the cathode must be at least equal to the sum of the peak currents drawn by all the electrodes.

Figure 1 gives a plot of saturation current as a function of cathode temperature in Kelvin.

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Fig. 1—Emisison current density vs. cathode temperature for several types of thermionic emitters. The shaded blocks at the bottom of the figure show the normal operating range for three of the cathodes. Curves are given for (A) The oxide-coated cathode. Curve A gives the saturation emission current density under pulsed conditions. Curve A gives the direct-current saturation emission density. The position of this curve may vary substantially with environmental conditions. Direct-current densities much in excess of 0.5 amp/cm² lead to a relatively short cathode life. (B) The pressed nickel, cathode. Curve B shows the direct-current saturation emission current density obtained from a pressed nickel cathode. (C) The impregnated nickel cathode. Curve C shows the saturation emission current obtained from the impregnated nickel cathode. The measurements were taken with 48-microsecond pulses and a repetition rate of 60 pulses per second. (D) Presared and impregnated tungsten cathodes. Curve D shows the saturation emission density obtained from pressed and impregnated tungsten cathodes based on $A = 2.5$ amp/cm² (K) and $\phi_e = 1.07$ electron-volts. (E) The thoriated-tungsten cathode. Curve E shows the measured saturation emission current density of an uncarburized thoriated-tungsten filament. (F) Tungsten filament. Curve F shows the saturation emission current density of a tungsten filament based on $A = 70$ amp/cm² (K) and $\phi_e = 1.5$ electron-volts. J. W. Gewartowski and H. A. Watson, "Principles of Electron Tubes," 1963: p. 48. Courtesy of D. Van Nostrand Company, Inc.
Table 1—Commonly Used Cathode Materials.

<table>
<thead>
<tr>
<th>Type</th>
<th>A</th>
<th>40</th>
<th>Efficiency (milliamperes/watt)</th>
<th>Specific Emission (amp/cm²)</th>
<th>Emisivity (watts/cm²)</th>
<th>Operating Temperature (°K)</th>
<th>Resistance Ratio (hot/cold)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bright tungsten (W)</td>
<td>70</td>
<td>4.50</td>
<td>5-10</td>
<td>0.25-0.7</td>
<td>70-84</td>
<td>2500-2600</td>
<td>14/1</td>
</tr>
<tr>
<td>Thoriated tungsten (Th-W)</td>
<td>4</td>
<td>2.65</td>
<td>40-100</td>
<td>0.5-3.0</td>
<td>25-28</td>
<td>1500-2000</td>
<td>10/1</td>
</tr>
<tr>
<td>Tantalum (Ta)</td>
<td>37</td>
<td>4.12</td>
<td>10-20</td>
<td>0.5-1.2</td>
<td>48-69</td>
<td>2380-2480</td>
<td>5/1</td>
</tr>
<tr>
<td>Oxide coated (Ba-Ca-Sr)</td>
<td>*</td>
<td>1.0-1.3</td>
<td>50-150</td>
<td>0.5-2.5</td>
<td>3-5</td>
<td>1000-1150</td>
<td>2.5 to 5.5/1</td>
</tr>
<tr>
<td>Impregnated</td>
<td>2.4</td>
<td>1.65</td>
<td>1.8-5.4</td>
<td>2.6-3.8</td>
<td>1300-1400</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The Richardson-Dushman equation does not apply to a composite surface of this type.

function of temperature for several types of emitters in common use.

Thoriated-tungsten and oxide-coated emitters should be operated close to specified temperature. A customary allowable heating-voltage deviation is ±5 percent. Bright-tungsten emitters may be operated at the minimum temperature that will supply required emission as determined by power output and distortion measurements. Life of a bright-tungsten emitter is lengthened by lowering the operating temperature. Figure 2 shows a typical relationship between filament voltage and temperature, life, and emission.

Mechanical stresses in filaments due to the magnetic field of the heating current are proportional to $I^2$. Current flow through a cold filament should be limited to 150 percent of the normal operating value for large tubes, and 250 percent for medium types. Excessive starting current may easily warp or break a filament.

Secondary Emission

When the surface of a solid is bombarded by charged particles having appreciable velocity, electrons are emitted from the solid. This is the process of secondary emission,* the most important ease being when the bombarding particles are also electrons. One then differentiates between incident and emitted electrons by the terms primary and secondary, respectively. The latter term commonly describes all electrons collected from a secondary emitter; these electrons comprise three groups: (A) true secondaries, (B) inelastically reflected primaries, and (C) elastically reflected primaries. True secondaries are considered to be those of the solid which have been excited above the energy level required for escape across the surface barrier. The three groups are separable to a degree on the basis of energy as indicated in the energy distribution curve of Fig. 3. True secondaries constitute the bulk of emitted electrons at moderate primary energies and have a mode energy of at most a few electron-volts. Their distribution is almost independent of primary energy. Electrons in the relatively flat interval within a, b constitute a mixture of true secondaries and inelastically reflected primaries. It has become customary to arbitrarily designate those emitted electrons having energies less than 50 electron-volts as true secondaries. Total secondary yield $\delta$, defined as the ratio of secondary to primary electron current, is independent of primary current but strongly dependent on primary energy as indicated in Fig. 4. The shape of the yield curve follows from generating and escape mechanisms; the former leading to an initial rise in yield with primary energy, and the latter causing an eventual reduction owing to increased penetration of primaries and a greater mean depth of escape of secondaries. Significant points of the yield curve are first and second crossover at which yield becomes unity, the maximum yield $\delta_{max}$, and the primary energy $E_m$ at which the maximum occurs. For most insulators, the first crossover occurs between 16 and 25 eV primary energy. Insulators generally exhibit higher yields than conductors, a property attributed to the absence of conduction electrons which tend to reduce the mean energy and the escape probability of secondaries through collision losses within the solid. The yield of insulators decreases noticeably as temperature is increased, owing to increasing electron-phonon interaction.

Secondary yield increases with angle of primary incidence, the effect being most pronounced at high primary energies. Yield is also a function of surface structure and may be minimized by employing physical trapping such as provided by a porous surface. Lowest yields are obtained for porous carbon deposits and highest yields for single crystal insulators having low electron affinity. Secondary yield may also be influenced by internal electric fields which tend to assist or retard the escape of secondaries. If such fields are strongly dependent on charge transport within the bombarded material, yield may become dependent on primary energy which can in turn give rise to anomalous time-dependent effects. Barring such effects, it appears that the interaction time for the secondary-emission process is of the order of 10^-12 second.

When the rate of bombardment by primary electrons becomes very low, as in single-electron counting, the statistical nature of the secondary-emission process becomes evident. The probability of obtaining $n$, $1$, $2$, $3$, etc., secondaries per incident primary is given by the Poisson distribution.

Commonly used secondary-emission materials are silver-magnesium or copper-beryllium alloy processed to provide a high-yield partly conductive surface film. Typically such surfaces exhibit yields of 2.5 to 4 at 100 eV primary energy.

Secondary emission is employed advantageously in the operation of many electron devices, such as camera tubes, storage tubes, and image intensifiers. A most important application lies in secondary-emitter multiplication, which provides a means for amplifying very weak electron currents as in photomultiplier tubes. A conventional electron multiplier consists of a number of secondary-emitting dynodes operated at progressively higher potentials and terminated by an electron-collecting electrode. Electrons incident on the first dynode are multiplied, the resultant secondaries are accelerated to the second dynode where the process is repeated, and so on throughout the multiplier structure.

If the dynodes exhibit uniform yield characteristics, the overall amplification $G$ obtained from a multiplier having $n$ dynodes is

$$G = p^n$$

where $p$ is the gain per dynode. The actual gain $p$ may be slightly less than the secondary yield $\delta$ because of multiplier geometry. In the absence of appreciable space-charge effects, maximum gain is realized when the available potential is uniformly distributed across the dynode chain. An empirical relation for $p$ may be obtained by approximating...
\[ g_{\text{max}} = e^{a} \]
\[ n_{\text{max}} = A^{\text{max}} (V/e) \]

where \( g_{\text{max}} \) and \( n_{\text{max}} \) are the optimum values of gain/dynode and number of dynodes, respectively, for maximizing overall amplification, given a total potential \( V \) available for distribution across the dynodes. The number of dynodes \( n \) is taken as the closest integer to the calculated value. In some cases, deviation from a uniform potential distribution and optimum gain conditions is advantageous, for example to reduce space-charge effects and improve time-delay and time-dispersion properties. Increasing the energy of electrons incident on the first dynode improves signal-to-noise ratio and single-electron-counting capability.

In addition to the conventional discrete dynode multiplier, a number of novel arrangements have been devised including crossed-field strip multipliers and tubular multipliers which commonly employ a continuous semiconductive dynode surface for potential distribution and multiplication. Tubular multipliers, when formed into a parallel array of small-diameter elements, may be employed for electron image intensification. Another form of multiplier commonly used for this purpose is the transmission secondary-emission multiplier, wherein secondaries exit from the side opposite primary incidence. The structure normally takes the form of a thin-film or porous supported layer, having the side of primary incidence made electrically conductive.

### Field Emission

If an electric field of sufficient magnitude is offered to the surface of a metal, the potential barrier at the surface will be lowered, allowing the escape of electrons, and field emission\(^*\) will result. The current has been found to vary with the applied field in accordance with

\[ J = C E^{2} \exp \left( -D / E \right) \text{ amperes/centimeter}^{2} \]

where \( J \) is the current density, \( E \) is the electric field at the surface, and \( C \) and \( D \) are approximately constant coefficients with \( D \) determined mainly by the work function. Field emission must be taken into account in the design of high-voltage tubes and apparatus, and is a factor in the operation of cold-cathode gas tubes. Although development is being carried on, there has been little use made yet of field emission in high-vacuum tubes.

### Photoemission

If photons with sufficient energy impinge on a photocathode, electrons are emitted.\(^*\) Such electrons are known as photoelectrons. For an input flux of fixed relative spectral distribution, the number of photoelectrons is proportional to the intensity of the input flux while the energy of the photoelectrons is independent of this intensity. The maximum energy of emitted electrons expressed in volts \( V \) depends on the wavelength \( \lambda \) and temperature. At absolute zero, according to Einstein’s law

\[ e(V + \phi) = h \nu / \lambda \]

where \( e \) = electron charge = 1.60 \times 10^{-19} \text{ coulomb} \]
\( \phi \) = work function in volts, \( h \) = Planck’s constant = 6.63 \times 10^{-27} \text{ joule-second} \]
\( v \) = velocity of light = 3 \times 10^{8} \text{ meters/second} \]
\( \lambda \) = wavelength in meters.

If a threshold wavelength \( \lambda_{0} \) is defined by

\[ e\phi = h c / \lambda_{0} \]

\( V \) is seen to be zero (except for thermal velocities) at the wavelength \( \lambda_{0} \). For \( \lambda > \lambda_{0} \), there is no photodetector emission at absolute zero. At temperatures above absolute zero there is always a finite probability of some photoemission at all wavelengths due to the thermalization of the electron distribution.

### Photocathode Response to Monochromatic Radiation

The output current \( dI \) in amperes, generated by a photocathode subjected to a monochromatic input flux \( d\nu \) in watts, is given by

\[ dI = s_{\nu} d\nu \]

where \( s_{\nu} \) is the monochromatic radiant sensitivity (or responsivity) of the photocathode in amperes/watt defined by this equation. Similarly, the number of electrons/second \( d\nu \) generated by an input flux \( d\nu \) in flux units is given by

\[ dN_{\nu} = q_{\nu} d\nu \]

where \( q_{\nu} \) is the monochromatic quantum efficiency of the photocathode in electrons/photon defined by this equation.

The monochromatic radiant sensitivity \( s_{\nu} \) is related to the monochromatic quantum efficiency \( q_{\nu} \) by

\[ q_{\nu} = eN_{\nu} / \epsilon_{\nu} = 8.08 \times 10^{8} \lambda_{0} / \lambda_{\nu} \]

Typical values of the monochromatic radiant sensitivity \( s_{\nu} \) and corresponding monochromatic quantum efficiency \( q_{\nu} \) as a function of wavelength \( \lambda \) are shown in Fig. 5 for some commonly used photocathodes, designated by their JEDEC registered "S" numbers. Table 2 gives typical peak sensitivities for the various surfaces, while Table 3 indicates the general composition and other properties of the common surfaces.

### Table 2—Typical Peak Photocathode Sensitivities

<table>
<thead>
<tr>
<th>S</th>
<th>( s_{\nu} ) max (amp/watt)</th>
<th>( \lambda_{\nu} ) max (nano-meters)</th>
<th>( q_{\nu} ) max (electron/photon)</th>
<th>( \lambda_{\nu} ) max (nano-meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.0025</td>
<td>800†</td>
<td>0.051</td>
<td>770†</td>
</tr>
<tr>
<td>S2</td>
<td>0.0010</td>
<td>420†</td>
<td>0.0058</td>
<td>400†</td>
</tr>
<tr>
<td>S3</td>
<td>0.0122</td>
<td>320†</td>
<td>0.13</td>
<td>350†</td>
</tr>
<tr>
<td>S4</td>
<td>0.0522</td>
<td>200†</td>
<td>0.21</td>
<td>300†</td>
</tr>
<tr>
<td>S5</td>
<td>0.0021</td>
<td>360†</td>
<td>0.0082</td>
<td>350†</td>
</tr>
<tr>
<td>S6</td>
<td>0.023</td>
<td>300†</td>
<td>0.056</td>
<td>400†</td>
</tr>
<tr>
<td>S7</td>
<td>0.021</td>
<td>450†</td>
<td>0.061</td>
<td>420†</td>
</tr>
<tr>
<td>S8</td>
<td>0.018</td>
<td>450†</td>
<td>0.14</td>
<td>400†</td>
</tr>
<tr>
<td>S9</td>
<td>0.018</td>
<td>500†</td>
<td>0.20</td>
<td>350†</td>
</tr>
<tr>
<td>S10</td>
<td>0.066</td>
<td>400†</td>
<td>0.07</td>
<td>300†</td>
</tr>
<tr>
<td>S11</td>
<td>0.021</td>
<td>400†</td>
<td>0.37</td>
<td>350†</td>
</tr>
</tbody>
</table>

† Neglecting short wavelength peak.

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Fig. 5—Typical absolute spectral response characteristics of photocathode devices.
TABLE 3—CHARACTERISTICS OF STANDARD PHOTOCEPHRODES.

<table>
<thead>
<tr>
<th>Photocathode Dark Current (nA/cm²)</th>
<th>25°C (lamp condition)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical 10⁻¹⁰</td>
<td>10⁻⁶</td>
</tr>
<tr>
<td>Typical 10⁻⁷</td>
<td>10⁻⁴</td>
</tr>
</tbody>
</table>

Typical Luminous Sensitivity (μA/lumen),

<table>
<thead>
<tr>
<th>Luminous Sensitivity (μA/lumen)</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical 6.5</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

TABLE 4—SPECTRAL MATCHING FACTORS.

<table>
<thead>
<tr>
<th>Source</th>
<th>λ₁</th>
<th>λ₂</th>
<th>S1</th>
<th>S11</th>
<th>S20</th>
<th>Photopic Eye</th>
</tr>
</thead>
<tbody>
<tr>
<td>2854°C K lamp</td>
<td>0</td>
<td>1200</td>
<td>0.52</td>
<td>0.060</td>
<td>0.112</td>
<td>0.071</td>
</tr>
<tr>
<td>5000°C blackbody</td>
<td>0</td>
<td>1200</td>
<td>0.53</td>
<td>0.26</td>
<td>0.34</td>
<td>0.140</td>
</tr>
<tr>
<td>Mean solar flux</td>
<td>0</td>
<td>1200</td>
<td>0.54</td>
<td>0.32</td>
<td>0.36</td>
<td>0.197</td>
</tr>
<tr>
<td>P1 phosphor</td>
<td>0</td>
<td>80</td>
<td>0.28</td>
<td>0.28</td>
<td>0.69</td>
<td>0.768</td>
</tr>
<tr>
<td>P4 phosphor</td>
<td>0</td>
<td>80</td>
<td>0.31</td>
<td>0.67</td>
<td>0.73</td>
<td>0.402</td>
</tr>
<tr>
<td>P11 phosphor</td>
<td>0</td>
<td>80</td>
<td>0.22</td>
<td>0.91</td>
<td>0.88</td>
<td>0.201</td>
</tr>
<tr>
<td>P20 phosphor</td>
<td>0</td>
<td>80</td>
<td>0.39</td>
<td>0.42</td>
<td>0.58</td>
<td>0.707</td>
</tr>
<tr>
<td>NaI(Tl)</td>
<td>0</td>
<td>80</td>
<td>0.53</td>
<td>0.88</td>
<td>0.90</td>
<td>0.046</td>
</tr>
</tbody>
</table>

Note: λ₁ and λ₂ are in nanometers.
the filter, \( J \) (no filter), is called the filter factor \( T(h, w, \alpha) \) and is given by

\[
T(h, w, \alpha) = \frac{\int_0^\infty \omega_0 \omega_1 \alpha \, d\alpha}{\int_0^\infty \omega_0 \omega_1 \alpha \, d\alpha}
\]

where \( \omega_0 \) is the transmission of the filter at a given wavelength \( \lambda \), and the notation \( T(h, w, \alpha) \) indicates that the filter factor is a function not only of the filter transmission \( h \), but also of the detector response \( w \) and the source distribution \( \alpha \). Typical filter factors are given in Table 5.

The ratio of emitted photocurrent with the filter, \( T(h) \), to the flux in lumens \( L(2854) \), incident on the filter (not on the cathode) from a 2854Â°C source is designated as \( S(\text{photocathode}+\text{filter}) \) and is given by

\[
S(\text{photocathode}+\text{filter}) = \frac{J}{J(\text{no filter})}
\]

and is expressed as \( S(\text{photocathode}+\text{filter}) = \frac{J}{J(2854)} \).

The magnitude of the luminous sensitivity, \( S(\text{photocathode}+\text{filter}) \), in amperes per lumen is used to specify cathode sensitivity, or more precisely, cathode-plus-filter sensitivity, over a selected spectral region, where the filter is chosen to restrict the flux incident on the photocathode to the desired region. The sensitivity, \( S(\text{photocathode}+\text{filter}) \), is then designated as the "infrared" sensitivity, or "red" sensitivity, or "blue" sensitivity, etc., depending on the predominant spectral region passed by the filter.

**ELECTRODE DISSIPATION**

After the electron stream has given up part of its useful component of its energy, the remainder is dissipated as heat in some suitable part of the tube. Five processes are commonly used to remove this heat. The amount which can be removed depends on the area available, the temperature differential, and, in the case of forced cooling, the coolant flow.

In computing cooling-medium flow, a minimum velocity sufficient to assure turbulent flow at the dissipating surface must be maintained. The figures for specific dissipations (Table 6) apply to clean cooling surfaces and may be reduced to a small fraction of the values shown by heat-insulating coatings such as scale or dust.

where \( P \) = power in watts, \( Q \) = flow in gallons per minute, and \( T_r, T_i \) = outlet and inlet water temperatures in degrees Kelvin, respectively.

This relationship is given in the nomogram of Fig. 6 with the temperature rise in degrees Fahrenheit or Celsius and the power in kilowatts.

**Radiation Cooling**

In a radiation-cooled system, that portion of the tube on which the heat is dissipated is allowed to reach a temperature such that the heat is radiated to the surroundings. The amount of heat which can be removed in this manner is given by

\[
P = 1089 Q\sqrt{(T_r/T_i) - 1}
\]

where \( Q \) = air flow in feet cubed per minute, other quantities as above.

**Evaporative Cooling**

A typical evaporative-cooled system consists of a tube with a specially designed anode immersed in a boiler containing distilled water. When power is dissipated on the anode, the water boils and the steam is conducted upward through an insulated pipe to a condenser. The condensate is then gravity fed back to the boiler, thus eliminating the pump required in a circulating water system.

For some transmitter applications the steam is directed downward to leave the space above the tube available for other components. Such a system requires a pump to return the condensate to the boiler, but even then the pump has to handle only about 0.05 of the amount of water required for a water-cooled system because of the exploitation of the latent heat of steam.

The size of the heat-exchanger equipment for an evaporative-cooled system is less than one-third of that required for a water-cooled system because of the greater mean temperature differential between the cooled liquid and the secondary coolant. Typical temperature differentials for the two systems are 75°C and 30°C, respectively.

The anode dissipation should not exceed 135 watts per square centimeter of external anode surface because at this point, often referred to as the "Leidenfrost" or "caldeation" point, the surface becomes completely covered with a sheath of vapor and the thermal conductivity between the anode and the cooling liquid drops to 30 watts per square centimeter, with resultant overheating of the anode. Special designs of the external anode surface (such as the "pineapple") allow up to 500 watts to be dissipated per square centimeter of internal anode surface.

**Forced Air Cooling**

With forced air cooling, a stream of air is forced past a suitable radiator. The heat which can be removed by this process is given by

\[
P = 1089 Q\sqrt{(T_r/T_i) - 1}
\]

**Table 6—Typical Operating Data for Common Types of Cooling**

<table>
<thead>
<tr>
<th>Type</th>
<th>Surface Temperature (°C)</th>
<th>Specific Dissipation of Cooling (watts/cm²)</th>
<th>Cooling-Medium Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation</td>
<td>400-1000</td>
<td>4-10</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>30-150</td>
<td>30-110</td>
<td></td>
</tr>
<tr>
<td>Forced air</td>
<td>150-200</td>
<td>0.5-1</td>
<td></td>
</tr>
<tr>
<td>Evaporative</td>
<td>100-120</td>
<td>90-125</td>
<td></td>
</tr>
<tr>
<td>Conduction</td>
<td>100-250</td>
<td>5-30</td>
<td></td>
</tr>
</tbody>
</table>
Conduction Cooling

When an external heat sink is available, heat may be removed from the tube by conduction. Since the electrode where the heat appears is usually at an elevated potential, it is often necessary to conduct the heat through an electrical insulation. Because of its relatively high thermal conductivity, beryllia ceramic can be used as a common insulator and thermal conductor between the anode of a tube and a heat sink.

Properties of beryllia:

Breakdown strength = 10 kV/mm
Dielectric constant = 6-8
Thermal conductivity = 2.62 watts/cm°C at 20°C, 1.75 watts/cm°C at 200°C
Dielectric loss factor = 4X10⁻⁶

The temperature drop in degrees Celsius across the beryllia ceramic is given by

\[ t_a - t_o = \frac{dW_a}{KA} \]  

(for a parallel configuration)

where \( t_a \) = temperature of tube anode (typical maximum 250°C), \( t_o \) = temperature of heat sink (typically 100°C), \( d \) = thickness of beryllia in cm, \( A \) = cross-sectional area of beryllia perpendicular to direction of heat flow, \( K \) = thermal conductivity of beryllia in watts/cm°C, and \( W_a \) = power dissipated on anode in watts.

To the temperature drop across the beryllia ceramic must be added the temperature drop across the interfaces between the ceramic and the anode and heat sink, typically 20°C for clamped surfaces at a loading of 25 watts/cm².

Because of its toxic nature, care must be taken in handling and disposal of beryllia ceramic.

Grid Temperature

Operation of grids at excessive temperatures will result in one or more serious effects: liberation of gas, high primary (thermal) emission, contamination of the other electrodes by deposition of grid material, and melting of the grid. Grid-current ratings should not be exceeded, even for short periods.

NOISE IN TUBES

There are several sources of noise in electron tubes, some associated with the nature of electron emission and some caused by other effects in the tube.

Shot Effect

The electric current emitted from a cathode consists of a large number of electrons and consequently exhibits fluctuations which produce tube noise and set a limit to the minimum signal that can be amplified. The root-mean-square value of the fluctuating (noise) component of the plate current \( I_p \) is given in amperes by

\[ I_{n}^2 = 2eI_{p} \Delta f \]

where \( I_p \) = plate direct current in amperes, \( e \) = electron charge = 1.6X10⁻¹⁹ coulomb, \( \Delta f \) = bandwidth in hertz, and \( I_{n} \) = space-charge reduction or smoothing factor. For temperature-limited cases, \( I_{n} = 1 \).

For space-charge-controlled regions

\[ I_{n}^2 = 2kT_{o}eI_{p}/\sigma l \]

where \( \sigma \) = Boltzmann's constant = 1.38X10⁻²³ joule/degree Kelvin, \( T_{o} \) = cathode temperature in degrees Kelvin, \( g \) = conductance or transconductance in mhos, which relates the output signal current to the input signal voltage, \( b \) = a factor which in most practical cases is nearly equal to its asymptotic value of \( 2(1 - (e/b)) = 0.644 \), and \( e \) = a tube parameter, related to the amplification factor and electrode spacings, which is unity for diodes and varies between 0.5 and 1.0 for negative-grid tubes.

Partition Noise

Excess noise appears in multielectrode tubes because of fluctuations in the division of the current between the different electrodes. In a grid-controlled tube, these fluctuations in current division reduce the effectiveness of the space-charge smoothing of the shot noise in the plate current. For a screen-grid tube, the root-mean-square noise currents in the cathode lead, the screen-grid lead, and the plate lead (\( I_{m}, I_{se}, \) and \( I_{s} \), respectively) are given by

\[ I_{n}^2 = 2I_{p} \Delta f \]

where \( I_{m} \) and \( I_{se} \) are the cathode and screen-grid currents, respectively.

Flicker Effect

The mechanism is not completely understood but appears to depend on the field distribution in the surface layer of the cathode due to its porous structure. Because this same field distribution will also influence the cathode activity and temperature, flicker noise will depend on cathode activity and temperature in a complicated manner.

The flicker noise spectrum is usually of the form \( f^{-1/2} \) with \( f \) close to unity and is important only at low frequencies. The sensitivity of audio, subaudio, and direct-current amplifiers is limited by the flicker noise generated in the first stage.

Collision Ionization

Free gas ions can be generated by collisions with the electron stream. The electrons thus liberated and collected by the anode will appear as noise in the anode circuit. The ions that travel to the cathode will travel slowly through the path...
potential minimum and reduce the space charge, which in turn will reduce the space-charge smoothing effect. This also will increase the noise in the anode circuit.

### Induced Noise

At high frequencies it is not necessary for electrons to reach an electrode for induced current to flow in the electrode leads. This noise is an important consideration in miniature tubes above 15 megahertz and becomes the principal limiting factor in low-noise amplifier design above about 100 megahertz. For microwave tubes, this is the dominant method by which beam noise is coupled to the output circuit.

### Miscellaneous Noise

Other noise may be present due to microphonics, hum, leakage, charges on insulators, poor contacts, and secondary emission.

### Evaluation of Tube Performance

There are two common ways of evaluating tube performance: equivalent noise input resistance value, and noise figure (or factor).

#### Equivalent Noise Input Resistance: A Resistor generates an amount of thermal noise that is equal to Johnson noise or Brownian motion noise.

\[
E_T = 4kT R_{eq} \delta f
\]

where \(E_T\) is the open-circuit root-meansquare fluctuating voltage measured across the resistor terminal in volts, \(T\) is temperature in degrees Kelvin, and \(R_{eq}\) is resistance in ohms. The equivalent noise input resistance in ohms \(R_{eq}\) is defined as that value of resistance which, when connected to the input of the tube and held at room temperature, will double the output noise power. This can be expressed as

\[
4kT R_{eq} \delta f^2 = 2kT \gamma^2 \delta f
\]

or

\[
R_{eq} = \frac{1}{T} \frac{1}{\gamma^2} \frac{2}{kT} \delta f^2
\]

where \(T = 293\text{K}\), \(\gamma^2\) is the effective space-charge reduction factor to include partition noise effects, and \(\gamma\) is the appropriate transconductance or conversion conductance as before. Practical approximations to \(R_{eq}\) are given in Table 8 for several tube functions.

#### Noise Figure: The noise figure of a tube is defined as the ratio of the available signal-to-noise ratio at the input to the signal-to-noise ratio at the output. It is usually given the symbol \(F\) and is always greater than unity. For a more-detailed discussion of noise figure refer to the chapter on "Radio Noise and Interference."

### Microwave Tubes

The noise appearing in the output circuit of a microwave tube is due in part to induced noise from the beam. Also, some of the electrons may be intercepted by the radio-frequency structure (microwave cavity, slow-wave circuit, etc.) giving rise to partition noise. In well-designed low-noise tubes, however, this latter effect is kept negligibly small.

For lossless linear beam tubes (traveling-wave amplifiers, klystron amplifiers, backward-wave amplifiers), the minimum obtainable noise figure \(F_{min}\) for one-dimensional single-velocity small-signal theory and high gain has been found to be given by

\[
F_{min} = 1 + (2\pi/kT_b) (S - \pi)
\]

where \(S = \pi\) is the basic noise parameter and is established in the region of the potential minimum of the beam. If certain assumptions concerning the potential minimum are made, such as full shot noise and uncorrelated current and velocity fluctuations, then values for \(S\) and \(\pi\) can be obtained. They are given as

\[
\pi = 0
\]

\[
S = (1 - (\pi/4))^{1/2} (kT_b/\pi)
\]

therefore

\[
F_{min} = 1 + (1 - \pi^{1/2}) (T_b/T_b)
\]

For \(T_b/T = 4\), \(F_{min} < 4\). The assumptions made are not entirely valid, as shown by the fact that noise figures of less than 4 have been obtained experimentally. At the present time values of \(S\) and \(\pi/S\) are obtained by measurement.

### LOW- AND MEDIUM-FREQUENCY TUBES

This section applies particularly to triodes and multigrid tubes operated at frequencies where electron-inertia effects are negligible. Traditionally the vacuum envelope of such tubes has been of glass with metal, usually copper, for the anode in larger sizes. In recent years the trend has been toward ceramic in place of glass for the external insulating portions of such tubes. Figure 7 shows a typical construction of a medium-power transmitting tube.

Ceramic-envelope tubes have the following advantages over glass tubes.

(A) The radio-frequency loss \(P_d\) in the seals of a tube is given by

\[
P_d = K f^2 R_b \delta f^3/2
\]

where \(K = \text{constant}, f = \text{frequency}, R = \text{resistivity of the conducting material}, \) and \(\mu = \text{permeability of the conducting material}. \) In glass-to-metal seals, the metal is normally of a magnetic material such as Kovar. As Kovar has high resistivity and permeability, the radio-frequency losses at the seals are therefore high, and at high frequencies cracking and/or glass sneak-in near the seals can result. With ceramic-to-metal seals this problem is minimized because the radio-frequency circulating currents at the seals flow through the metallizing and plating on the ceramic. The resistivity is low, and the permeability is unity.

(B) Ceramics have a lower dielectric loss than glass. Furthermore the loss factor of glass rapidly rises with temperature. This leads to a "runaway" condition, glass suck-in, and hence severe limitation of maximum frequency of operation of glass tubes.

(C) The safe operating temperature of a ceramic-to-metal seal may be between 290 and 250 degrees Celsius as against 180 degrees Celsius for Kovar glass seals.

(D) High bakeout temperature of ceramic-envelope tubes during evacuation increases reliability and life.

(E) Ceramic tubes withstand higher thermal and mechanical shocks than those with glass envelopes. They can also be manufactured to closer dimensional tolerances.

### Coefficients

**Amplification factor \(\mu\)**: Ratio of incremental plate voltage to control-electrode voltage change at a fixed plate current with constant voltage on other electrodes

\[
\mu = \frac{\delta V_p}{\delta V_c} \quad E_i, E_x = \text{constant}
\]

**Transconductance \(g_m\)**: Ratio of incremental plate current to control-electrode voltage change at constant voltage on other electrodes

\[
g_m = \frac{\delta I_p}{\delta V_c} \quad E_i, E_x = \text{constant}
\]

When electrodes are plate and control grid, the ratio is the mutual conductance \(g_m\)

\[
g_m = \mu T_p
\]

**Variational (AC) Plate Resistance \(r_p\)**: Ratio of incremental plate voltage to current change at constant voltage on other electrodes

\[
r_p = \frac{\delta V_p}{\delta I_p} \quad E_i, E_x = \text{constant}
\]

**Total (DC) Plate Resistance \(R_p\)**: Ratio of total plate voltage to current for constant voltage on the plate

\[
R_p = \frac{\delta V_p}{\delta I_p} \quad E_i, E_x = \text{constant}
\]
Table 9—Tube Characteristics for Unipotential Cathode and Negligible Saturation of Cathode

<table>
<thead>
<tr>
<th>Function</th>
<th>Parallel-Plane Cathode and Anode</th>
<th>Cylindrical Cathode and Anode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diode anode current (amperes)</td>
<td>( G \alpha e_0 )</td>
<td>( G \alpha e_0 )</td>
</tr>
<tr>
<td>Triode anode current (amperes)</td>
<td>( G (\alpha + \mu \rho) / (1 + \mu) )</td>
<td>( G (\alpha + \mu \rho) / (1 + \mu) )</td>
</tr>
<tr>
<td>Diode perseverance ( G_1 )</td>
<td>( 2.3 \times 10^{-4} (A_1 / \rho) )</td>
<td>( 2.3 \times 10^{-4} (A_1 / \rho \rho_n) )</td>
</tr>
<tr>
<td>Triode perseverance ( G_2 )</td>
<td>( 2.3 \times 10^{-4} (A_2 / \rho d_0) )</td>
<td>( 2.3 \times 10^{-4} (A_2 / \rho \rho_n r_0) )</td>
</tr>
<tr>
<td>Amplification factor ( \mu )</td>
<td>( 2.7 A_1 (d_0 / d_2 - 1) / (\rho \log (\rho / 2 r_2)) )</td>
<td>( (2.7 d_0 / \rho) (\log (d_0 / d_2)) / (\rho \log (2 r_2)) )</td>
</tr>
<tr>
<td>Mutual conductance ( \mu_n )</td>
<td>( 1.5 \mu (\rho / (\alpha + 1))(E_0) )</td>
<td>( 1.5 \mu (\rho / (\alpha + 1))(E_0) )</td>
</tr>
</tbody>
</table>

where \( A_1 \) = effective anode area in square centimeters, \( d_1 \) = anode-cathode distance in centimeters, \( d_2 \) = grid-cathode distance in centimeters, \( \beta \) = geometric constant, a function of ratio of anode-to-cathode radius; \( \beta = 1 \) for \( r_2 / r_1 > 10 \) (Fig. 9), \( s \) = pitch of grid wires in centimeters, \( r_2 \) = grid-cathode radius in centimeters, \( r_1 \) = anode radius in centimeters, \( r_0 \) = cathode radius in centimeters, and \( r_2 \) = grid radius in centimeters.

These equations are based on theoretical considerations and do not provide accurate results for practical structures; however, they give a fair idea of the relationship between the tube geometry and the constants of the tube.

High-Frequency Triodes and Multigrid Tubes*

When the operating frequency is increased, the operation of triodes and multigrid tubes is affected by electron-inertia effects. The design features that distinguish the high-frequency tube shown in Fig. 10 from the lower-frequency tube (Fig. 7) are: reduced cathode-to-grid and grid-to-anode spacings, high emission density, high power density, small active and inactive capacitances, heavy terminals, short support leads, and adaptability to a cavity circuit.

Factors Affecting Ultra-High-Frequency Operation

Electron Inertia: The theory of electron-inertia effects in small-signal tubes has been formulated; no comparable complete theory is now available for large-signal tubes.

When the transit time of the electrons from cathode to anode is an appreciable fraction of one radio-frequency cycle:

(A) Input conductance due to reaction of electrons with the varying field from the grid becomes appreciable. This conductance, which increases as the square of the frequency, results in lowered gain, an increase in driving-power requirement, and loading of the input circuit.

(B) Grid-anode transit time introduces a phase lag between grid voltage and anode current. In oscillators, the problem of compensating for the phase lag by design and adjustment of a feedback circuit becomes difficult. Efficiency is reduced in both oscillators and amplifiers.

(C) Distortion of the current pulse in the grid-anode space increases the anode-current conduction angle and lowers the efficiency.

Electrode Admittances: In amplifiers, the effect of cathode-lead inductance is to introduce a conductance component in the grid circuit. This effect is serious in small-signal amplifiers because the loading of the input circuit by the conductance current limits the gain of the stage. Cathode-grid and grid-anode capacitive reactances are of small magnitude at ultra-high frequencies. Heavy currents flow as a result of these reactances and tubes must be designed to carry the currents without serious loss. Coaxial cavities are often used in the circuits to resonate with the tube reactances and to minimize resistive and radiation losses. Two circuit difficulties arise as operating frequencies increase:

(A) The cavities become physically impossible as they tend to take the dimensions of the tube itself.

(B) Cavity \( Q \) varies inversely as the square root of the frequency, which makes the attainment of an optimum \( Q \) a limiting factor.

Scaling Factors: For a family of similar tubes, the dimensionless magnitudes such as efficiency are constant when the parameter \( \phi = \pi d / V^{\frac{1}{2}} \) is constant, where \( \pi \) = frequency in megahertz, \( d \) = cathode-to-anode distance in centimeters, and \( V \) = anode voltage in volts.

Based on this relationship and similar considerations, it is possible to derive a series of factors that determine how operating conditions will vary as the operating frequency or the physical dimensions are varied (Table 10). If the tube is to be scaled exactly, all dimensions will be reduced inversely as the frequency is increased, and operating conditions will be as given in the "Size-Frequency Scaling" column. If the dimensions of the tube are to be changed but the operating frequency maintained, operation will be as in the "Size Scaling" column. If the dimensions are to be maintained but the operating frequency changed, operating conditions will be as in the "Frequency Scaling" column. These factors apply in general to all types of tubes.
**Table 10—Scaling Factors For Ultra-High-Frequency Tubes.**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Ratio</th>
<th>Size-Frequency Scaling</th>
<th>Size Scaling</th>
<th>Frequency Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>$V_2/V_1$</td>
<td>1</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Field</td>
<td>$E_2/E_1$</td>
<td>$f$</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Current</td>
<td>$I_2/I_1$</td>
<td>1</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Current density</td>
<td>$j_2/j_1$</td>
<td>$f$</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Power</td>
<td>$P_2/P_1$</td>
<td>1</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Power density</td>
<td>$h_2/h_1$</td>
<td>$f$</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Conductance</td>
<td>$G_2/G_1$</td>
<td>1</td>
<td>$d$</td>
<td>$f$</td>
</tr>
<tr>
<td>Magnetic-flux density</td>
<td>$B_2/B_1$</td>
<td>$f$</td>
<td>$d$</td>
<td>$f$</td>
</tr>
</tbody>
</table>

$d =$ ratio of scaled to original dimensions  
$f =$ ratio of original to scaled frequency

---

**MICROWAVE TUBES**

The reduced performance of space-charge control tubes in the microwave region has fostered the development of other types of tubes for use as oscillators and amplifiers at microwave frequencies. Such tubes generally function on the basis of the modulation of the velocity of an electron stream rather than its density. They may be roughly divided simply into linear beam devices and cross-field devices. In the former the electron stream flows essentially linearly, often with a collimating magnetic field to counteract space-charge spreading; in the latter the electron stream follows a curved path under the action of orthogonal electric and magnetic fields. The linear beam devices are often referred to as O-type, while the cross-field devices are referred to as M-type.

**Terminology**

**Bunching:** Any process that introduces a radio-frequency conduction-current component into a velocity-modulated electron stream as a direct result of the variation in electron transit time that the velocity modulation produces.

---

**Pulse:** Momentary flow of energy of such short time duration that it may be considered as an isolated phenomenon.

**Pushing Figure of an oscillator** is the rate of frequency pushing in megahertz per ampere or megahertz per volt.

**Q:** The $Q$ of a specific mode of resonance of a system is $2\pi$ times the ratio of the stored electromagnetic energy to the energy dissipated per cycle when the system is excited in this mode.

**Reflector:** Electrode whose primary function is to reverse the direction of an electron stream. It is also called a repeller.

**Reflax Bunching:** Type of bunching that occurs when the velocity-modulated electron stream is made to reverse its direction by means of an opposing direct-current field.

**Slow-Wave Structure:** A microwave circuit, as used in beam-type microwave tubes, capable of propagating radio-frequency waves with phase velocities appreciably less than the velocity of light.

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**LINEAR BEAM TUBES**

The principal types of linear beam tubes are the klystron, the traveling-wave amplifier, and the backward-wave oscillator.

**Klystrons**

A klystron* is an electron tube in which the following processes may be distinguished:

(A) Periodic variations of the longitudinal velocities of the electrons forming the beam in a region confining a radio-frequency field.

(B) Conversion of the velocity variation into conduction-current modulation by motion in a region free from radio-frequency fields.

(C) Extraction of the radio-frequency energy from the beam in another confined radio-frequency field.

The transit angles in the confined fields are made short ($\phi < \pi/2$) so that there is no appreciable conduction-current variation while traversing them.
Several variations of the basic klystron exist. Of these, the simplest is the 2-cavity amplifier or oscillator. The most important of these is the reflex klystron, used as a low-power oscillator. The multicavity high-power amplifier is also important.

Two-Cavity Klystron Amplifier: An electron beam is formed in an electron gun and passed through the gap associated with the two cavities (Fig. 12). After emerging from the second gap, the electrons pass to a collector designed to dissipate the remaining beam power without the production of secondary electrons. In the first gap, the electron beam is accelerated and accelerated in succeeding half-periods of the radiofrequency cycle, the magnitude of the change in speed depending on the magnitude of the alternating voltage impressed on the cavity. The electrons then move in a drift space where there are no radio-frequency fields. Here, the electrons that were accelerated in the input gap during one half-cycle catch up with those that were decelerated in the preceding half-cycle, and a local increase of current density occurs in the beam. Analysis shows that the maximum of the current-density wave occurs at the position, in time and space, of those electrons that passed the center of the input gap as the field changed from negative to positive. There is therefore a phase difference of $\pi/2$ between the current wave and the voltage wave that produced it. At the end of the drift space, the initially uniform electron beam has been altered into a beam showing periodic density variations. This beam now traverses the output gap and the variations in density induce an amplified voltage wave in the output circuit, phased so that the negative maximum corresponds with the phase of the bunch center. The increased radio-frequency energy has been gained by conversion from the direct-current beam energy. The 2-cavity amplifier can be made to oscillate by providing a feedback loop from the output to the input cavity, but a much simpler structure results if the electron beam direction is reversed by a negative electrode, termed the reflector.

Reflex Klystron*: A schematic diagram of a reflex klystron is shown in Fig. 13. The velocity-modulation process takes place as before, but analysis shows that in the recharging field used to reverse the direction of electron motion, the phase of the current wave is exactly opposite to that in the 2-cavity klystron. When the bunched beam returns to the cavity gap, a positive field extracts maximum energy from the beam, since the direction of electron motion has now been reversed. Consideration of the phase conditions shows that, for a fixed cavity potential, the reflex klystron will oscillate only near certain discrete values of reflector voltage for which the transit time measured from the gap center to the reflection point and back is given by

$$v_{tr} = 2\pi (N + \frac{1}{2})$$

where $N$ is an integer called the mode number.

By varying the reflector voltage around the value corresponding with the mode center, it is possible to vary the oscillation frequency by a small percentage. This fact is made use of in providing automatic frequency control or in frequency-modulation transmission.

Reflex-Klystron Performance Data: The performance data for a reflex klystron are usually given in the form of a reflector-characteristic chart. This chart displays power output and frequency deviation as a function of reflector voltage. Several modes are often displayed on the same chart. A typical chart is shown in Fig. 14.

There are two rather distinct classes of reflex klystrons in current large-scale manufacture (Table 11).

(A) Low-power tubes suitable for use in local-oscillator service, maser pumping, antenna-pattern testing, or similar applications. These have power outputs in the range of 10 milliwatts to 1.2 watts. Typically, for local-oscillator service, a power of 10 to 100 milliwatts is necessary to operate crystal mixers with the required degree of isolation. For such applications as antenna testing or pumping of cryogenically cooled masers, power of 500 milliwatts to 2 watts is usually required. The electronic tuning range required is about 50 megahertz independent of center frequency, but the linearity of the $\Delta f$ versus $\Delta V$ characteristic is relatively unimportant.

(B) Tubes as frequency modulators in microwave links. These usually require considerably greater power, up to about 10 watts, and the linearity of the $\Delta f$ versus $\Delta V$ characteristic over a limited (for example 10-megahertz) excursion is of primary importance as this parameter determines the harmonic margins in the system. Second-harmonic margins of -90 decibels for deviations of 125 kilohertz have been observed; the third-harmonic margins are about -120 decibels.

Multicavity Klystrons: Multicavity klystrons have been perfected for use in various different fields of application: applications requiring extremely high power levels and continuous-wave systems in which moderate powers (tens of kilowatts) are required. An example of the first application is a power source for nuclear-particle accelerators, where ultra-high-frequency television and tropos국 Scatter transmitters are examples of the latter.

A multicavity klystron amplifier is shown schematically in Fig. 15. The example shown has 3 cavities all connected in parallel. The radiofrequency input modulates the beam as before. The bunched beam induces an amplified voltage across the second cavity, which is tuned to the operating frequency. This amplified voltage recharges the beam with a certain phase shift and the more strongly bunched beam excites a greatly amplified wave in the output circuit.

![Fig. 12—Two-cavity klystron amplifier. J. W. Groenendaal and H. A. Watson, "Principles of Electron Tubes," 1964; p. 296. Courtesy of D. Van Nostrand Company, Inc.](image)

![Fig. 13—Schematic of reflex klystron with power supply](image)

![Fig. 14—Klystron reflector characteristic chart. Courtesy of Sperry Gyroscope Company.](image)

![Fig. 15—Three-cavity klystron amplifier.](image)

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### Table 11: Classes of Reflex Klystrons

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Power Output (mW)</th>
<th>Useful Mode Width $\Delta f_{max}$ (MHz)</th>
<th>Operating Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 000</td>
<td>150</td>
<td>40</td>
<td>300</td>
</tr>
<tr>
<td>5 000</td>
<td>40</td>
<td>40</td>
<td>350</td>
</tr>
<tr>
<td>24 000</td>
<td>35</td>
<td>120</td>
<td>750</td>
</tr>
<tr>
<td>35 000</td>
<td>&gt;15</td>
<td>50</td>
<td>2000</td>
</tr>
<tr>
<td>50 000</td>
<td>10-20</td>
<td>60-140</td>
<td>600</td>
</tr>
</tbody>
</table>

---

![Image for Table 11](image)
It is found that the optimum power output is obtained when the second cavity is slightly detuned. Moreover, when increased bandwidth is required, the second cavity may be loaded with a resonant load in order to increase gain. Modern multi-cavity klystrons use magnetically focused high-power beams, and under these conditions high gains, large power outputs, and reasonable values of efficiency are readily obtained.

Continuous-wave multivavity klystrons are available with outputs of around 10 kilowatts at frequencies up to 5000 megahertz. The efficiencies are of the order of 30 percent and the gains vary between 20 and 50 decibels, according to the number of cavities, bandwidth, etcetera. Pulsed tubes have been designed for outputs of 50 megawatts and with efficiencies of over 40 percent at frequencies near 3000 megahertz.

**Traveling-Wave Tubes**

The traveling-wave tube differs from the klystron in that the radio-frequency field is not confined to a limited region but is distributed along a wave-propagating structure. A longitudinal electron beam interacts continuously with the field of a wave traveling along this wave-propagating structure. In its most common form it is an amplifier, although there are related types of tubes that are basically oscillators.

The principle of operation may be understood by reference to Fig. 16. An electron stream produced by an electron gun, travels along the axis of the tube, and is finally collected by a suitable electrode. Spaced closely around the beam is a circuit, in this case a helix, capable of propagating a slow wave. The circuit is proportioned so that the phase velocity of the wave is small with respect to the velocity of light. In typical low-power tubes, a value of the order of one-tenth of the velocity of light is used; for high-power tubes the phase velocity may be two or three times higher. Suitable means are provided to couple an external radio-frequency circuit to the slow-wave structure at the input and output. The velocity of the electron stream is adjusted to be approximately the same as the axial phase velocity of the wave on the circuit.

When a wave is launched on the circuit, the longitudinal component of its field interacts with the electrons traveling in approximate synchronism with it. Some electrons will be accelerated and some decelerated, resulting in a progressive re-arrangement in phase of the electrons with respect to the wave. The electron stream, thus modulated, in turn induces additional waves on the helix. This process of mutual interaction continues along the length of the tube with the net result that direct-current energy is given up by the electron stream to the circuit as radio-frequency energy, and the wave is thus amplified.

By virtue of the continuous interaction between a wave traveling on a broad-band circuit and an electron stream, traveling-wave tubes do not suffer the gain-bandwidth limitation of ordinary types of electron tubes. By proper circuit design, such tubes are made to have bandwidths of an octave or more, and even more in many cases. The helix is an extremely useful form of slow-wave circuit because the impedance that it presents to the wave is relatively high and because, when properly proportioned, its phase velocity is almost independent of frequency. The essential feature of this type of tube is the approximate synchronism between the electron stream and the wave. For this reason, the traveling-wave tube will operate correctly over only a limited range in voltage. Practical considerations require that the operating voltages be kept as low as is consistent with obtaining the necessary beam input power; the voltage, in turn, determines the phase velocity of the circuit. The electron velocity in centimeters/second is determined by the accelerating voltage \( V \) in accordance with

\[
p = 5.53 \times 10^{12} \sqrt{V}.\]

Figure 17 shows a typical relationship between gain and beam voltage. The gain \( G \) of a traveling-wave tube is given approximately by

\[
G = \frac{1}{1 + BCN}\]

where \( A \) is the initial loss due to the installation of the modes on the helix and lies in the range from -6 to -9 decibels, \( B \) is a gain coefficient that accounts for the effect of circuit attenuation and space charge, \( C \) is a gain parameter that depends on the impedances of the circuit and the electron stream \( \left( \frac{K_f}{a_e} \right) \times 10^N \), \( K_f \) is beam voltage, \( N \) is number of active wavelengths in tube = \( 1/\lambda_0 \), \( \lambda_0 \) is free-space wavelength, \( \kappa \) is phase velocity of wave along tube, and \( \epsilon \) is velocity of light. The term \( K_f/2 \left( a_e \right) \) is a normalized wave impedance which may be defined in a number of ways.

In practice, the attenuation of the circuit will vary along the tube, and consequently the beam gain per unit length will not be constant. The total gain will be a summation of the gains of various sections of the tube.

Commonly, \( C \) is of the order of 0.02 to 0.2 in helix traveling-wave tubes. The gain of low- and medium-power tubes varies from 20 to 70 decibels with 20 decibels being a common value. The gain in a tube designed to produce appreciable power will vary somewhat with signal level when the beam voltage is adjusted for optimum operation.

Figure 18 shows a typical characteristic.

To restrain the physical size of the electron stream as it travels along the tube, it is necessary to provide focusing fields, either magnetic or electrostatic, of a strength appropriate to overcome the space-charge forces that would otherwise cause the beam to spread. Until fairly recently a longitudinal magnetic field supplied by a solenoid electromagnet was used for this purpose. Continuing demands for improved efficiency and reliability, and for size and weight reduction, however, have forced the development of permanent magnet-type focusing structures. At the present time, traveling wave focusing structures produced by permanent magnet structures is rapidly becoming predominant.

Several techniques for electrostatic containment of the electron stream have also been developed. There have been a variety of helical slow-wave structures where an appropriate voltage difference between the helices provides, in effect, a distributed Einzel lens. Because of voltage breakdown problems as well as increased power supply requirements, these techniques has not yet proved practical for use in linear beam tubes.

Other types of slow-wave circuits in addition to the helix are possible, including a number of periodic structures. In general, such designs are capable of operation at higher power levels but at the expense of bandwidth.

**Traveling-Wave-Tube Performance Data**

Traveling-wave tubes are designed to emphasize particular inherent characteristics for specific applications. Three general classes are distinguished.

- **Low-Noise Amplifiers:** Tubes of this class are intended for the first stage of a receiver and are proportioned to have the best possible noise figure. This requires that the random variations in the electron stream be minimized and that steps be taken also to minimize the intrinsic noise. Tubes have been made for commercial use with noise figures as low as 3 decibels in S-band and 6 decibels or less over the entire range from 1000 to 12000 megahertz. Gains of from 20 to 35 decibels are typical at low frequencies. An amplifier not more than a few milliwatts. Performance of this order can be achieved with either permanent-magnet or electromagnet focusing structures. Recently a new class of tubes has been developed which offers medium-power performance (1 to 5 watts) with reasonable low-noise performance (noise figures of 10 to 14 decibels).

- **Intermediate Power Amplifiers:** These tubes are intended to provide power gain under conditions where neither noise nor large values of power output are important. Gains of 20 or more decibels are customary and the maximum output power is usually in the range of 100 milliwatts to 1 watt.

- **Power Amplifiers:** For this class of tubes, the application is usually the output stage of a transmitter; the power output, either continuous-wave
or pulsed, is of primary importance. Much active development continues in this area and the values of power obtainable are steadily increasing. At present, continuous-wave power ranges from tens of kilowatts in the ultra-high-frequency region to more than 100 watts at 10,000 megahertz. Tubes especially designed for pulsed operation provide considerably higher power. Several megawatts of peak power have been achieved at 3000 megahertz. Efficiencies in excess of 30 percent have been obtained and this may be further enhanced by recently developed collector-depression techniques. Power gains of 20 to 30 decibels are normal.

**Backward-Wave Oscillators**

A member of the traveling-wave-tube family, the O-type backward-wave oscillator,* makes use of the interaction of the electron stream with a radio-frequency circuit wave whose phase and group velocities are 180° apart. The group velocity, and thus the direction of energy flow, is directly opposed to the direction of electron motion. Figure 19 shows schematically a backward-wave tube with connection to both ends of the slow-wave structure, so that operation as either oscillator or amplifier could be achieved. An electron beam is produced by the electron gun, traverses the slow-wave structure, and is dissipated in the collector structure. During its transit the beam is confined by a longitudinal magnetic field. With a beam current of sufficient magnitude, the beam-structure interaction will produce oscillations and microwave power will be delivered from the end of the structure adjacent to the electron gun. At beam-current levels below the "start-oscillation" value, a radio-frequency signal may be introduced at the collector end of the device and the tube will operate as an amplifier.

To improve interaction efficiency, electron beams with hollow cross sections are usually used. This places all the electrons as close as possible to the slow-wave structure in the region of maximum radio-frequency field. The reason here is that the strength of the 1-space harmonic field goes to zero on the axis. To produce this hollow-cross-section beam it is necessary to confine the magnetically confined electron flow from the cathode, and thus the electron gun is entirely immersed in the magnetic field.

**Electron-Beam Parametric Amplifiers**

Parametric amplification occurs through a time-varying or nonlinear parameter of the system. A simple mechanical example is a child pumping up a swing. He gives energy to the swing amplitude by raising and lowering his center of gravity at twice the swing frequency. The time-varying parameter is the effective length of the swing.

Table 13—Performance of Typical Low-Power Backward-Wave Oscillators.

<table>
<thead>
<tr>
<th>Frequency Range (GHz)</th>
<th>Tuning Voltage (V)</th>
<th>Cathode Current (mA)</th>
<th>Minimum Power Output (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0-2.0</td>
<td>250-1150</td>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>2.0-4.0</td>
<td>300-1800</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>4.0-8.0</td>
<td>250-2400</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>5.3-11.0</td>
<td>245-2400</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>8.0-12.4</td>
<td>550-2400</td>
<td>10</td>
<td>25</td>
</tr>
</tbody>
</table>

*Electron-beam parametric amplifiers are, for the most part, linear beam (O-type) devices, although crossed-field (M-type) devices have been built.* The usual O-type amplifier, such as the traveling-wave tube, uses the coupling between a circuit and the slow space-charge wave on the beam to obtain gain. Since this wave carries negative kinetic power, it is not possible to couple the noise on the wave out of the beam. For parametric amplification, it is necessary to use either the fast space-charge wave or the fast cyclotron wave. Because these waves carry positive kinetic power, it is possible (theoretically) to couple all the noise on these waves out of the beam.

A block diagram of an O-type parametric amplifier is shown in Fig. 20. A microwave tube version of this block diagram is shown in Fig. 21. This device has 20-decibel gain at 410 megahertz with a bandwidth of 67 megahertz and a double-channel noise figure of 2.4 decibels. The tube uses the fast
**Cyclovoltmeter, wave, and the input and output are microwave forms of the Cuckoo coupler.**

**Crossed-Field Tubes**

The earliest type of crossed-field tube was the magnetron oscillator. The carontron and the crossed-field amplifier have been developed more recently. Crossed-field tubes generally operate with higher conversion efficiencies than linear beam devices, making them especially attractive for high-power applications.

**Magnetrons**

A magnetron is a high-vacuum tube containing a cathode and an anode, the latter usually divided into two or more sections. A constant magnetic field modifies the space-charge distribution and the current-voltage relations. In modern usage, the term “magnetron” refers to the magnetron oscillator in which the interaction of the electronic space charge with the resonant system converts direct-current power into alternating-current power, usually at microwave frequencies.

Many forms of magnetrons have been made in the past and several kinds of operation have been employed. The type of tube that is now almost universally employed is the multicavity magnetron generating traveling-wave oscillations. It possesses the advantages of good efficiency at high frequencies, capability of high outputs either in pulsed or continuous-wave operation, moderate magnetic-field requirements, and good stability of operation. A section through the basic anode structure of a typical magnetron is shown in Fig. 22.

In magnetrons, the operating frequency is determined by the resonant frequency of the separate cavities arranged around the central cylindrical cathode and parallel to it. A high direct-current potential is placed between the cathode and the cavities, and radio-frequency output is brought out through a suitable transmission line or waveguide usually coupled to one of the resonator cavities. Under the action of the radio-frequency voltages across these resonators and the axial magnetic field, the electrons from the cathode form a bunch of space-charge cloud that rotates around the tube axis, exciting the cavities and maintaining their radio-frequency voltages.

**Magnetron Performance Data**

The performance data for a magnetron are usually given in terms of two diagrams, the performance chart and the Riek diagram.

**Performance Chart:** This is a plot of anode current along the abscissa and anode voltage along the ordinate of rectangular-coordinate paper. For a fixed typical tube load, pulse duration, pulse-repetition rate, and setting of the tuning of the resonant tubes, lines of constant magnetic field, power output, efficiency, and frequency may be plotted over the complete operating range of the tube. Regions of unsatisfactory operation are indicated by cross hatching. For tunable tubes, it is customary to show performance charts for more than one setting of the tuning. In the case of magnetrons with attached magnets, curves showing the variation of anode voltage, efficiency, frequency, and power output with change in anode current are given. A typical chart for a magnetron having 8 resonators is given in Fig. 23.

**Riek Diagram:** This shows the variation of output, anode voltage, efficiency, and frequency with changes in the voltage standing-wave ratio and phase angle of the load for fixed operating conditions such as magnetic field, anode current, pulse duration, pulse-repetition rate, and the setting of the tuning of the resonant tubes. The Riek diagram is plotted on polar coordinates, the radial coordinate being the reflection coefficient, the angle coordinate the phase of the standing wave at the voltage standing wave, minimum from a suitable reference plane on the output terminal. On the Riek diagram, lines of constant frequency, anode voltage, efficiency, and output may be drawn (Fig. 24).

Magnetrons for pulsed operation have been built to deliver peak powers ranging from 3 megawatts at 3000 megahertz to 10 kilowatts at 3000 megahertz. Continuous-wave magnetrons having output ratings ranging from 1 kilowatt at 3000 megahertz to a few watts at 30,000 megahertz have been produced. Operation efficiencies up to 60 percent at 3000 megahertz are obtained, falling to 30 percent at 30,000 megahertz.

**Carcinotron**

The carcinotron is a U-type backward-wave oscillator in which the electron stream traverses the tube and interacts with the fields on the slow-wave structure under conditions where the electric and magnetic fields are perpendicular to each other. Figure 25 shows schematically a linear version of the carcinotron. In the electron gun, current is drawn from the cathode when the accelerator voltage is applied. Because of the presence of the magnetic field, directed as shown, the electron paths are curved approximately 90° so that they enter the interaction region between the slow-wave structure and the sole. If the voltages and magnetic field strength are proper, the electrons will travel along a path approximately parallel to the structure until they reach the collector.

Although Fig. 25 shows a linear arrangement, carcinotrons are conventionally designed in a circular arrangement to conserve magnet size and weight. In this arrangement, the sole approximates the appearance of the cathode of a magnetron and the slow-wave structure is in the position of the magnetron anode, but neither the sole nor the structure are re-entrant.

The carcinotron performance is similar to that of the O-type backward-wave oscillator but it offers several of the advantages of crossed-field devices. High-efficiency operation is possible with

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![Fig. 22—Magnetron oscillator.](image)

![Fig. 23—Performance chart for pulsed magnetron.](image)

![Fig. 24—Riek diagram.](image)

![Fig. 25—Linear version of an M-carcinotron oscillator.](image)
values of 20 to 30 percent being readily obtained. This efficiency capability makes the caecinotron useful as a high-power device with continuous-wave capabilities of hundreds of watts through X-band. Its construction is such as to permit direct feeding to very high frequencies, with several milliwatts of power having been achieved at frequencies beyond 300 gigahertz.

The caecinoton, like the O-type backward-wave oscillator, is voltage-tunable with the oscillation frequency being approximately directly proportional to the cathode slow-wave-structure voltage. This linear relationship simplifies the associated electronic tuning circuit considerably. Frequency pushing is also considerably lower than in O-type backward-wave oscillators. The M-type caecinoton has the disadvantage, however, that it is relatively noisy, with spurious power output often not more than 10 to 15 decibels below the main signal output.

In addition to obvious usage as high-power tunable signal sources, caecinotrons with their high noise may be used as electronic countermeasure jamming sources.

Crossed-Field Amplifiers

Crossed-field amplifiers as a general class of tubes are mechanically quite similar to the crossed-field oscillator or caecinotron. As may be seen from Fig. 26, the major difference is that the slow-wave structure is not re-entrant whereas in the magnetron both the beam and the circuit are re-entrant.

Referring to Fig. 26, voltage and magnetic field are applied as for the magnetron. A radio-frequency signal is applied to the structure and progresses in a clockwise direction toward the output terminal. Current spokes, produced in the cathode-circuit region by the radio-frequency electric fields, also progress in a clockwise direction synchronously with the circuit wave. The interaction between the beam and circuit wave results in a growing of the circuit wave and thus gain. If desired, interaction with a backward mode may also be accomplished with this device.

Since the beam is re-entrant, the crossed-field amplifier will oscillate if the circuit gain becomes high. Gain is usually limited to 10 to 15 decibels. If only a portion of the circumference is used for the slow-wave structure and a drift area is left between the two ends of the structure, the feedback mechanism is disrupted and gains of 15 to 20 decibels may be realized.

![Schematic drawing of a crossed-field amplifier.](image)

**Table 13—Ionization Properties of Gases**

<table>
<thead>
<tr>
<th>Gas</th>
<th>Ionization Energy (volts)</th>
<th>Collision Probability $P_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>24.5</td>
<td>12.7</td>
</tr>
<tr>
<td>Neon</td>
<td>21.5</td>
<td>17.5</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>16.7</td>
<td>37.0</td>
</tr>
<tr>
<td>Hydrogen (H₂)</td>
<td>15.9</td>
<td>20.0</td>
</tr>
<tr>
<td>Argon</td>
<td>15.7</td>
<td>34.5</td>
</tr>
<tr>
<td>Carbon monoxide</td>
<td>14.2</td>
<td>23.8</td>
</tr>
<tr>
<td>Oxygen</td>
<td>13.5</td>
<td>34.5</td>
</tr>
<tr>
<td>Krypton</td>
<td>13.3</td>
<td>45.4</td>
</tr>
<tr>
<td>Water vapor</td>
<td>13.2</td>
<td>55.2</td>
</tr>
<tr>
<td>Xenon</td>
<td>11.5</td>
<td>62.5</td>
</tr>
<tr>
<td>Mercury</td>
<td>10.4</td>
<td>67.0</td>
</tr>
</tbody>
</table>

The power output of the crossed-field amplifier is essentially independent of the radio-frequency drive signal and it thus operates as a saturated amplifier. This characteristic makes it unsuitable for amplifying amplitude-modulated signals.

Crossed-field amplifiers offer the advantage of relatively high efficiency, 40 to 60 percent or even higher, and they may be designed to provide very high peak output powers. Their disadvantages are their low gain, limited bandwidth, high noise, and saturated-amplifier characteristic.

**GAS TUBES**

**Ionization**

A gas tube is an electron tube in which the pressure of the contained gas is such as to affect substantially the electrical characteristics of the tube. Such effects are caused by collisions between moving electrons and gas atoms. These collisions, if of sufficient energy, may dislodge an electron from the atom, thereby leaving the atom as a positive ion. The electron space charge is effectively neutralized by these positive ions and comparatively high free-electron densities are easily created.

Table 13 gives the energy in electron-volts necessary to produce ionization. The column $P_c$ is the kinetic-theory collision probability per centimeter of path length for an electron in a gas at 16°Celcius at a pressure of 1 millimeter of mercury. The collision frequency is given by

$$f_c = \frac{e^2}{2\pi m_k}$$

where $f_c$ = collisions per second, $P_c$ = collision probability in collisions per centimeter per millimeter of pressure, and $p$ = gas pressure in millimeters of mercury.

**Characteristics of Gas Tubes**

Gas tubes may be generally divided into two classes, depending on whether the cathode is hot or cold and thus on the mechanism by which electrons are supplied.

**Hot Cathode Gas Tubes:** The electrons in the hot-cathode gas tube are produced thermionically. The voltage drop across such tubes is that required to produce ionization of the gas and is generally a few tens of volts. The current conducted by the tube depends primarily on the emission capability of the cathode. Figure 27 shows the effect of the ionized gas on the voltage distribution in a hot-cathode tube.

**Cold Cathode Gas Tubes:** The electrons in a cold-cathode tube are produced by bombardment of the cathode by ions and/or by the action of a localized high electric field. The voltage drop across such a tube is higher than in the hot-cathode tube because of the mechanism of electron generation, and the current which can flow is limited. Figure 28 shows the effect of tube geometry and gas pressure on the voltage required to initiate the discharge.

Figure 28 shows a typical volt-ampere characteristic of a cold-cathode discharge. Cold-cathode gas tubes may be divided into two categories, depending on the region of this characteristic in which they operate.

**Glow Discharge Tubes** require a drop of several hundred volts across the tube and operate in region I. The current is of the order of tens of millamperes.

**Arc Discharge Tubes** operate in region II. They are not, strictly speaking, cold-cathode, for the current is drawn from a localized spot on the cathode which is consequently heated and provides a large thermionic current. The voltage drop is thus lowered. Such a tube is capable of conducting currents of thousands of amperes at voltage drops of tens of volts. Mercury-arc cathodes are used.
Hydrogen Thyatrons are hot-cathode hydrogen-filled triodes designed for use as electronic switching devices where short anode delay time is important. In pulsing service they are capable of switching tens of megawatts at voltages of kilovolts. Anode delay time and time jitter are in the nanosecond range, and the tubes do not depend on ambient temperature for proper operation. Hydrogen thyatrons are also used in crowbar applications to protect other devices from fault voltages or currents and are capable of handling peak currents of several-thousand amperes.

Triggered Spark gaps are cold-cathode gas tubes operating in the arc discharge regime III. The gaps contain two high-power electrodes and a trigger electrode which is generally fired through a step-up pulse transformer by a simple low-energy pulse. The gaps are used as electronic switching devices for peak currents of tens of thousands of amperes and voltages of tens of kilovolts. They can discharge stored energy of several thousand joules and are used for energy transfer in exploding wire-generator circuits, plasma discharge ignition, spark chambers, and Kerr cells. They are also used in crowbar applications for fast-acting protection of other circuit components against fault voltages and currents. Before conduction, the gap possesses a high capacitance and a very high resistance to the circuit. After triggering, when the gap is conducting, the impedance drops to a few ohms or less.

Voltage Regulators of the glow discharge type take advantage of the volt-ampere characteristic in region II, where the voltage is nearly independent of the current. They operate at milliamperes and up to a few hundred volts.

Voltage regulators of the corona discharge type operate at currents of less than a milliamperes and at voltages up to several thousand volts.

Microwave Applications of Gas Tubes

Noise Sources: Gas discharge devices possess a highly stable and repeatable effective noise temperature when in the fired condition. This feature provides a convenient and accurate means for determining noise figures. The microwave energy radiated from a gas discharge plasma is coupled into a radio-frequency transmission line with which it is used. The amount of radio-frequency power available from a gas discharge tube depends mainly on the nature of the gas fill, the geometric characteristics of the discharge tube, and the electron temperature of the positive column or plasma. The design parameter which most strongly determines the noise temperature is the type of gas employed. Any of the noble gases may be used in a noise source. In practice, however, only two or three are normally used:

- Gas: $F = \text{ENR (dB)}$
- Helium: 21.0
- Neon: 15.5
- Argon: 15.3

When referring to a noise source or generator, the ratio of its noise power output to thermal noise power is called the Excess Noise Ratio (ENR).

$$F = \text{ENR} = \frac{\left( \frac{T_e}{T_0} - 1 \right) - 1}{\left( \frac{T_e}{T_0} - 1 \right)}$$

where $T_e$ = ratio of the noise output power of the receiver with the noise generator on, to that with the noise generator off; $T_0$ = temperature in degrees K; $T_e$ = effective noise temperature (in degrees K) of the noise generator in the fired condition. The expression $\left( \frac{T_e}{T_0} - 1 \right)$ is termed the excess noise power of the noise source. When $T_1 = \text{ENR} = 290^\circ\text{K}$

$$\text{ENR}_1 = \frac{\left( \frac{T_e}{T_0} - 1 \right) - 1}{\left( \frac{T_e}{T_0} - 1 \right)}$$

ENR (dB) = 10 log$_e$ ENR.

The effective temperature source is equal to the temperature of the discharge only if the coupling of the transmission line to the discharge is complete. Otherwise there is a reduction in the noise power output which can best be determined by measuring the fired and un-fired insertion loss of the unit at the frequencies of interest. The relation between these factors is given by

$$\left( \frac{T_e}{T_0} - 1 \right) - 1 = \left( \frac{L_e}{L_0} \right)$$

where $\left( \frac{T_e}{T_0} - 1 \right)$ is the effective excess noise power of the generator, $\left( \frac{T_e}{T_0} - 1 \right)$ is the excess noise power, and $L_e$ and $L_0$ are the insertion losses in the fired and un-fired conditions, respectively. This correction should be subtracted from the apparent measured noise figure.

Noise figures are always measured with reference to a standard temperature of 290°K $(T_0)$. If the ambient temperature $(T_i)$ of the noise-generator termination differs from the standard temperature, the noise figure calculated must be corrected. To find the correction factor, substitute the ambient temperature of the noise-generator termination for $T_0$ in the following equation and add the temperature factor $(T/F_i)$ to the noise figure calculated.

$$F = \frac{1}{\left( \frac{1}{Y - 1} \right) - \left( \frac{T_i}{T_0} - 1 \right)}$$

TR Tubes: Transmit-receive tubes are gas discharge devices designed to isolate the receiver section of radar equipment from the transmitter during the period of high power output. A typical TR tube and its circuit are illustrated in Fig. 31. The cones in the waveguide form a transmission cavity tuned to the transmitter frequency and the tube conducts received low-power-level signals from the antenna to the receiver. When the transmitter is operated, however, the high-power signal causes gas ionization between the cone tips, which detunes the structure and reflects all the transmitter power to the antenna. The receiver is protected from the destructively high level of power and all the available transmitter power is useful output.

**Microscope Gas Discharge Circuit Elements:** Because of the high free-electron density, the plasmas of gas discharges are capable of strong interaction with electromagnetic waves in the microwave region. In general, microwave phase shift and/or absorption result. If used in conjunction with a magnetic field, these effects can be increased and made nonreciprocal. Phase shift is a result of the change in dielectric constant caused by the plasma according to

$$\varepsilon_e = \varepsilon_e (1 - \frac{\text{en}}{\text{en} + 1})$$

where $\varepsilon_e$ = dielectric constant in plasma, $\varepsilon_e$ = dielectric constant in free space, $\text{en}$ = electron density in electrons/cm$^3$, and $f_s$ = signal frequency in megahertz.

Absorption of microwave energy results when electrons, having gained energy from the electric field of the signal, lose energy in collisions with the tube envelope or neutral gas molecules. This absorption is a maximum when the frequency of collisions is equal to the signal frequency and the absolute magnitude is proportional to the microwave electron density.


**Light-Sensing and -Emitting Tubes**

**Radiometry and Photometry**

Radiometric and photometric systems are generally based on the concept of radiated flux, where flux is defined as the total amount of radiation passing through a unit area per unit time. If a flux is measured in terms of its thermal heating ability, the most common unit is the watt and the resultant measurement system is called radiometry. If a flux is measured in terms of its ability to stimulate the standard photopic human eye, the resultant unit is the lumen, and the resultant measurement system is called photometry. A third choice for the measurement of flux is the number of photons per unit time. These three choices, in conjunction with the MKS system of units, lead to the three mutually compatible systems of units shown in Table 14. Table 15 gives equivalents between units in different photometric measurement systems.

**Flux Units:** The number of lumens dL, and the number of photons per second dN, associated with a monochromatic flux dW, in watts are given by

\[ dL = 680 \, El \, dW \]

and

\[ dN = (\lambda/hc) \, dW \]

where 680 = number of lumens per watt of radiated energy at the peak photopic eye response, E0 = normalized (to unity maximum) photopic human eye response (Fig. 32), \( \lambda \) = wavelength of the monochromatic radiation (meters), h = Planck's constant \( = 6.62 \times 10^{-34} \) (joule-second), and c = velocity of light \( = 3.0 \times 10^{10} \) (meters per second).

The number of lumens L and the number of photons per second N between the wavelength limits of \( \lambda_1 \) and \( \lambda_2 \) are given by

\[ L = 680 \int_{\lambda_1}^{\lambda_2} E_{\lambda} \lambda \, d\lambda \]

and

\[ N = (\lambda/hc) \int_{\lambda_1}^{\lambda_2} E_{\lambda} \lambda \, d\lambda \]

where

\[ \lambda = \omega_{\text{max}} \int_{\lambda_1}^{\lambda_2} \omega_{\lambda} \, d\lambda \]

and where \( \omega_{\text{max}} \) = maximum spectral density in watts per unit wavelength in the spectral band between \( \lambda_1 \) and \( \lambda_2 \), and \( \omega_{\lambda} \) = relative spectral distribution of the radiation source on a thermal-energy basis, normalized to a maximum value of 1. Some typical \( \omega_{\lambda} \) spectral distributions are shown in Fig. 32.

**Optical Imaging:** In an optical lens system of flux-gathering diameter \( D_f \) in meters, focal length \( f \) in meters, and optical transmittance \( T \), the ratio \( f/D_f = n \) is called the f-number of the lens. If the source is an object of radiance or luminance \( B \) in flux units per steradian per meter \( ^2 \) is imaged by this system with a linear magnification \( m \), and assuming Lambertian emittance characteristics, then for the solid angle subtended by the optical system, the image will be subjected to an irradiance \( I_I \) or luminance \( I_L \) in flux units per meter \( ^2 \) given by

\[ I_I = \frac{\pi BT}{4\mu n^2 (m+1)^2 + m^2} \]

For objects at infinity, \( m = 0 \), and

\[ I_L = \frac{\pi BT}{4\mu n^2} \]

The irradiance (or luminance) \( I_L \) in flux units per meter \( ^2 \) is allowed to fall on a nonabsorbing Lambertian diffusing surface, the resultant image radiant energy (or luminance) \( B_r \) in flux units per steradian per meter \( ^2 \) is given by

\[ \beta B_r = I_L \]

Any desired size of measuring flux units, such as watts, lumens, or photons/second (Table 14), can be selected for expressing the object radiant energy (or luminance) \( B \) in flux units steradian-\( ^1 \) meter-\( ^2 \) and the irradiance (or luminance) \( I_L \) in flux units meter-\( ^2 \) in these relationships. Thus, a radiation \( B \) in watts steradian-\( ^1 \) meter-\( ^2 \) would be paired with an irradiance \( I_L \) in watt meter-\( ^2 \), a luminance \( B_r \) in lumens steradian-\( ^1 \) meter-\( ^2 \) with an illuminance \( I_L \) in lumen meter-\( ^2 \), and a radiation \( B \) in photon second-\( ^1 \) steradian-\( ^1 \) meter-\( ^2 \) with an irradiance \( I_L \) in photon second-\( ^1 \) steradian-\( ^1 \) meter-\( ^2 \).

Any spectral distribution modifications, if present, would be included in the numerical magnitude of the lens transmission \( T \), defined as the ratio of the total output flux from the optical system to the corresponding input flux. Selection of appropriate alternative pairs of luminaire and illuminance units when the flux units are not explicitly stated (first column of Table 15) must be made with care. Thus, candle centimeter-\( ^2 \) (or stilb) would be paired with phot, candle meter-\( ^2 \) (or nits) with lux, and candle foot-\( ^2 \) with footcandle. Even greater difficulty arises when the factor \( \Pi \) in the preceding relationships is absorbed or included in the units of luminance. Thus the product \( \Pi B \) in candela would be paired with \( I_L \) in lux, the product \( \Pi B \) in lumen with.
In phot, the product $I_B$ in millinewtrem with $I_L$ in milliamp, and the product $I_B$ in footcandles with $I_L$ footcandles. These difficulties are avoided by the use of the compatible systems of radiation units shown in Table 14.

**Typical Approximate Illumination Values at the Earth's Surface:**
- Sun at zenith: $10^4$ footcandles
- Full moon: $3 \times 10^3$ footcandles
- Reading fine print: $10^2$ footcandles
- November football field: $5 \times 10^3$ footcandles
- Surface of moon seen from Earth: $1.5 \times 10^3$ footcandles
- Summer baseball field: $3 \times 10^2$ footcandles
- Crater of carbon arc: $4.5 \times 10^4$ footcandles
- Sun seen from Earth: $5.2 \times 10^6$ footcandles

**Photoconductivity**

Photoconductivity is the increase in electrical conductivity of a material which takes place when the material is illuminated with infrared, visible, or ultraviolet light.

The absorption of light is a quantum process in which electrons are excited to higher energy levels. Ordinarily, the excited electrons are more mobile than unexcited electrons. Photoconductivity is commonly analyzed in terms of the number and mobility of the excited electrons in an electron conduction band and of electron vacancies or "holes" in a lower-energy valence band. To maintain a steady current, both types of carriers must be generated in the volume of the material, or else charge carriers must enter the photocathode at one of the electrodes. Many photocathodes make "ohmic" contacts with their electrodes. These serve as practically unlimited reservoirs of mobile electrons, free to enter the photocathode volume. Even in these photocathodes the steady dark current is usually limited to a low value by a built-up of a space-charge-potential barrier in the photocathode.

At the same time that mobile photocathodes are excited (thermally or optically) in the interelectrode volume, positive charges must also be generated; these compensate the charge of the photoelectrons in such a way that a "photocurrent" can be superimposed on the small space-charge-limited "dark" current originating at the electrode. If the positive charges are immobile, then long after the photocathode has passed through the photocoatector into the anode, the immobile positive charges may remain to support a photocurrent of electrons, drawing on the reservoir of electrons at the cathode. This will continue until the immobile "holes" or impurity centers are neutralized by recombination with some of the mobile electrons. Since the recombination lifetime may be much longer than the electron transit time between electrodes, the number of "photoelectrons" transported across the photocoatector may be much larger than the rate of generation of photocathodes in the photocoatector volume. This ratio of photocurrent to generation rate is called the photocoatector gain.

The photoconductivity gain of a pure material can often be greatly increased by addition of localized traps lying near the conducting band. Since these are in thermal equilibrium with the conducting band, they serve as an additional reservoir of the charge carriers. This can increase both the response time and the sensitivity by a large factor.

Practically all materials are photocathodes in the sense that light of the correct wavelengths will generate current carriers. However, in many materials the photoconductivity is not detectable by ordinary measurements, either because of very short carrier lifetimes or because of a large dark current. The useful photocathodes, characterized by comparatively long lifetimes and low dark current, have most of their charge carriers immobile (in the dark). Light of the proper energy can excite these carriers through the forbidden energy regions into the conduction bands. The long-wavelength limit of photoconductivity at low temperatures is approximately given by

$$\lambda_{max} = \frac{h \nu}{E_f}$$

where $E_f$ is the forbidden band gap, $h$ is Planck's constant, and $\nu$ is the velocity of light. For wavelength longer than 8 microns, this equation gives a band gap smaller than 1 volt. Photocathodes with such small energy gaps are usually cooled to reduce the dark conductivity due to thermal excitation of carriers across the gap.

A commonly used figure of merit for photocathodes is the detectivity $D^*$, defined as the signal-to-noise ratio at a given chopping frequency with an amplifier bandwidth of 1 hertz for a photodiode of 1 square centimeter, divided by the light flux in watts. Detectivities for several typical photocathodes at room temperature are shown in Fig. 33. The photocathodes with long-
Phosphors

Fluorescent screens are used in various electronic devices such as image tubes, cathode-ray tubes, and storage cathode-ray tubes to convert electron energy into radiant energy. These viewing screens are comprised of many small-diameter (2 to 3 microns) phosphor crystals which emit light when bombarded by high-energy electrons. The spectral response of a phosphor screen is determined by its chemical and physical composition, deposition methods, and the tube processing procedures. Phosphor screens with given output characteristics have been categorized and assigned type numbers. Typical absolute spectral-response characteristics of aluminized phosphor screens are shown in Fig. 34.

**Phosphor Efficiency**

Over a rather wide range of magnitudes, the ratio \( U/g \) of total radiated energy

\[
U \text{ into a } 2\pi \text{ solid angle, to exciting electron charge } g \text{ for typical aluminized phosphor screens, Fig. 35, is independent of excitation time, beam-current magnitude, and bombarded area.}
\]

Consequently, the ratio \( W/I \) of average total radiated flux \( W \) to average exciting current \( I \) is also independent of the magnitude of the average current \( I \), the peak current \( I_{\text{peak}} \), and the bombarding area, and may therefore be used to describe the flux-generating properties of raster-scanned as well as steady-state excited phosphor screens. Experimentally, it is found that the ratio \( W/I \) is not a linear function of electron beam voltage \( V \), as might be expected if the phosphor converts the beam energy \( IV \) linearly into radiant flux, but behaves in general as shown in Fig. 36, with an offset energy component \( IV \), followed by an approximately linear dependence on the added energy \( IV(V-V_{\text{max}}) \) over the usual working voltage range, \( V_{\text{min}} < V < V_{\text{max}} \).

The flux-generating properties of a phosphor screen behaving in the above manner are therefore approximately described by

\[
W/I = \frac{e_0}{e_1(V-V_{\text{c}})}, \quad V_{\text{c}} < V < V_{\text{max}}
\]

where \( W \) = average radiated flux (watts), \( I \) = average exciting electron beam current (amperes), \( e_0 \) = dimensionless phosphor efficiency ratio \( [\text{radiated watts per exciting electron beam watts (watts watt}^{-1}] \), \( V \) = electron beam voltage, \( V_{\text{c}} \) = extrapolated offset knee voltage (Fig. 36), and \( V_{\text{c}} < V < V_{\text{max}} \) = limits of operating electron beam voltage.

The offset knee voltage \( V_{\text{c}} \) for typical aluminized phosphor screens has a magnitude between 1 and 3 kilovolts, and may be attributed primarily to electron beam power losses in penetrating the aluminizing layer plus possible inert-phosphor-particle coatings.

The average monochromatic radiant flux output per unit wavelength \( d\lambda \) from a phosphor screen at wavelength \( \lambda \) under these conditions is given by

\[
d\lambda = \int_0^{e_1} e_0(V-V_{\text{c}}) d\lambda
\]

where

\[
e_0 = \int_0^{\infty} e_0(V-V_{\text{c}}) d\lambda
\]

\[
e_{\text{max}} = \text{maximum spectral power density efficiency}[\text{watts meter}^{-1} \text{ watt}^{-1}]
\]

\[
w_{\text{n}} = \text{normalized relative power density of the radiant energy spectrum.}
\]

Typical values of the “spectral efficiency” \( e_0 \) are plotted in Fig. 34 for a number of commonly
radiation in lumen watt$^{-1}$ at the peak eye-response wavelength, and $E_s$=relative photopic eye response.

Some typical values of the dimensionless "spectral-matching-factor ratio"

$$\int_0^\infty \omega \lambda E_\lambda d\lambda = \int_0^\infty \omega v \lambda E_v d\lambda$$

appearing in these relationships are given in Table 4.

If the total flux output is measured in photons second$^{-1}$ instead of watts or lumens, then the corresponding relationships are

$$N/N_\ast = Y(V - V_\circ)$$

$$d\lambda N = v \lambda d\lambda$$

$$Y = \int_\infty^\infty \omega v \lambda d\lambda$$

where $N$=total number of output photons, $N_\ast$=total number of triggering input electrons, $Y$="quantum yield factor" (photons electron$^{-1}$), $d\lambda N$=number of photons per unit wavelength (photons meter$^{-1}$), and $y_\ast$=spectral efficiency (photons meter$^{-1}$) electron$^{-1}$ (shown on curved coordinate scales of Fig. 34).

The "quantum yield factor" $Y$, tabulated in Fig. 34 for the listed typical phosphor behavior, is useful in predicting the quantum yield $N/N_\ast$ of a phosphor screen for an effective exciting electron beam voltage $V - V_\circ$ according to the above equation.

For a phosphor screen radiating according to Lambert's Cosine Law (often approximately, but not exactly, valid), the radiation $B$ in (watts ster$^{-1}$) meter$^{-2}$ and the luminance $L_\lambda$ in (lumens meter$^{-2}$) watt$^{-1}$ are related to their corresponding absolute values $v$ and $E_\lambda$ by

$$E_\lambda = 680 E_\lambda v$$

and

$$E_\lambda = 680 E_\lambda v \int_0^\infty \omega v \lambda E_v d\lambda$$

where 680=luminous equivalent of monochromatic

**LIGHT-SENSING TUBES**

**Image Tubes and Image Intensifiers**

An image tube* is an optical-image-in-to-optical-image-out electron tube device, combining an input

---

*Note: The image contains technical details and formulas related to phosphor screens, radiation, and quantum efficiency. It requires a background in physics and electronics to fully understand.
where $I_{s}$ is input image irradiance on the photocathode in watt meter$^{-2}$ at the wavelength $\lambda$ and $m$ is differential magnification ratio of the image tube -- output incremental image size divided by the corresponding input incremental image size.

For a spectrally distributed input flux having a known relative spectral distribution $w_\lambda$ and a known radiated power $P_{\text{rad}}$, in waves between the wavelength limits $\lambda_1$ and $\lambda_2$, the resulting total output flux $W_\lambda$ in waves exiting from the image tube is given by

$$W_\lambda = \int_{\lambda_2}^{\lambda_1} \int_{0}^{\infty} w_\lambda \, d\lambda \, d\phi \times \alpha(G_{\text{rad}}, \lambda) \times \alpha(ISD, V - V_\lambda) \times \alpha(ISD, V - V_\lambda) \times \alpha(ISD, V - V_\lambda)$$

where $w_\lambda_{\text{max}}$ is peak radiant sensitivity of the input photocathode in amperes per watt, $\alpha_{\text{rad}}$ is relative radiant sensitivity of the input photocathode as a function of wavelength $\lambda$ normalized to unity maximum, $\alpha_{\text{G}}$ is relative spectral distribution of the power density spectrum of the input flux normalized to unity maximum, and $\alpha_{\text{ISD}}$ is wattage gain of the image tube for the relative spectral distribution $w_\lambda$ and the wavelength limits $\lambda_1$ and $\lambda_2$. Typical values for the magnitude of the dimensionless spectral-matching-factor ratio

$$\left( \int_{0}^{\infty} w_\lambda \, d\lambda \right) \int_{\lambda_1}^{\lambda_2} w_\lambda \, d\lambda$$

are found in Table 4.

The total output flux $L_\lambda$ in lumens exiting from an image tube, corresponding to the total output flux $W_\lambda$ in waves, can be computed from the flux conversion relationships given in the section on Radiometric and Photometric, or from the following relationship

$$L_\lambda = \int_{\lambda_2}^{\lambda_1} \int_{0}^{\infty} E_\lambda \, d\lambda \, d\phi \times \alpha(G_{\text{rad}}, \lambda) \times \alpha(ISD, V - V_\lambda) \times \alpha(ISD, V - V_\lambda) \times \alpha(ISD, V - V_\lambda) = G_{\text{L}}$$

where $G_{\lambda}$ is luminous gain of the image intensifier tube -- ratio of the output flux to the power dissipated in the particles of the output phosphor screen (Fig. 34), $E_\lambda$ is relative output radiation in volt watts, $E_\lambda$ is relative output radiation in volt watts, and $E_\lambda$ is extrapolated knee voltage of the output phosphor screen (Fig. 36).

If the phosphor screen radiates flux according to Lambert's law (equation approximately valid), the corresponding output image radiance $R_\lambda$ in watt steradian$^{-1}$ meter$^{-2}$ is given by

$$R_\lambda = G_{\lambda} \, f_{\lambda} \, E_{\lambda} \, V_{\text{rad}}$$

where $V_{\text{rad}}$ is extrapolated knee voltage in volts for the sandwich phosphor screen, and $gamma$ is optical coupling efficiency of the sandwich electrode -- ratio of the flux falling onto the sandwich photocathode to the output flux emitted by the sandwich phosphor screen. Typical values of the two integrals appearing in this relationship are given in Table 4.

The combination of a phosphor screen and a photoconverter to produce current gain $G_\lambda$ can also be achieved by operating the output flux from one or more image tubes into a second stage of a second stage

Resolution in image tubes and image intensifier tubes is a subject parameter describing the number of pairs of equally spaced illuminated and unilluminated bars per unit distance at the photocathode imaged onto the input photocathode surface which can just be distinguished visually by a trained observer under stated test conditions.

Distortion is a parameter describing any change in the geometric shape of the output image compared with the input image. Radial distortion results when image magnification leads to "pinching" distortion, radially decreasing magnification leads to "barrel" distortion, and radially changing image rotation leads to "spher" distortion.

Vacuum Photodiodes

The combination of a photocathode and an anode electrode for collecting the emitted photocurrent in an evacuated envelope is called a vacuum photodiode. A positive anode potential sufficient to assure collection of all emitted photoelectrons (that is, "saturate" the diode photocathode) is normally required, the tube then acting as a constant-current generator (Fig. 37). The power supply potential $V_a$ must assure sufficient anode potential in the presence of a voltage drop in the load resistor $R_\lambda$.

Under these conditions the total anode output current $I_{\text{an}}$ neglecting all noise fluctuations, is given by

$$I_{\text{an}} = I_{\text{em}} + I_\gamma + I_{\text{S}}$$

where $I_{\text{em}}$ is emitted photocathode signal current, $I_\gamma$ is emitted photocathode photocurrent due to stray background flux, $I_{\text{S}}$ is emitted photocathode thermona.
Gas Photodiodes

In diode phototubes not containing a high vacuum, ionization by collision of electrons with neutral molecules may occur so that more than one electron per incident photon is emitted. The "gas amplification factor" has a value of between 3 and 5; a higher factor causes instabilities. Gas-tube operation is restricted to frequencies below about 10,000 hertz.

Photomultipliers

The combination of a photoemissive cathode and a secondary-emission electron multiplier is called a photomultiplier.* Emitted photoelectrons from the photocathode are directed under the influence of a suitable electric field, often called the "focus field," to the surface of a secondary-emitting electrode, the "first dynode." Subsequently emitted secondary electrons, increased in number by the effective secondary-emission ratio $e_1$, are then directed to the secondary-emitting surface of a subsequent dynode by an appropriate electric field for further multiplication. Continuing this process for a succession of dynodes and collecting the multiplity charge produced at the output electrode, a so-called "anode" or "collector," leads to a charge or current amplification $G$ given by

$$G = e_n$$

for $n = e_1 e_2 ... e_n$

where $e_n$ is the secondary-emission ratio.

Output Current: Disregarding all noise fluctuations of the output current, and assuming operation within the usual linear-response region, a photomultiplier acts as a constant-current source generating an output current $I_0$ given by

$$I_0 = \frac{G e_n I_0}{V}$$

for $0 < I < T$

and

$$I_0 = \frac{(2QR/RC)}{(1/RC)} + \exp[-(1/RC)] - 1$$

for $T < I < \infty$

where $Q$ is the peak input signal or the peak input signal due to an incident flux $\phi$, and $R$ is the load resistance.

Signal and Noise: Noise fluctuations of the output current in photomultipliers can be divided into two classes: dark noise, occurring in the absence of input flux; and noise-in-signal, including "quantum" noise resulting from the inherent quantum limit of the input flux as well as uncontrolled fluctuations of that flux. The presence of an appreciable, in fact often predominant, noise-in-signal current component in photomultipliers depending on the instantaneous signal current magnitude requires emphasis in applying noise concepts to photomultipliers, and may lead to erroneous conclusions regarding photomultiplier behavior, particularly for a modulated flux input.

Photomultipliers as Scintillation and Single-Electron Counters

In combination with suitable scintillating material, typically thallium-activated NaI crystals, photomultipliers are extensively used to detect the single flashes of light generated by the scintillation produced by a single nuclear disintegration (a gamma ray) or a single ionizing particle, typically a word current from a nuclear disintegration process. The scintillating material generates an average of $N$ photons per disintegration incident on the photocathode of a peak quantum efficiency $Y_{max}$, and the resultant average charge pulse $Q_{av}$ appearing in the anode circuit (disregarding all photons or electrons producing no output charge) will be given by

$$Q_{av} = N Y_{max} e_n$$

where $e_n$ is the noise factor for the photomultiplier, and $Y_{max}$ is the maximum photomultiplier gain. As well as in the number of effective photons $N$ generated by the scintillator, the anode charge $Q_{av}$ will vary in magnitude within certain practical limits. The determination of the average magnitude of $N$, which in turn is used to determine the energy of the triggering input particle, for example the gamma ray. The ratio of the spread of the amplitude of individually observed values of the charge $Q_{av}$
of electrons passes through the aperture. These electrons impinge on the secondary-emitting surface of a dynode where, on the average, they give rise to 3 to 6 secondary electrons. These secondary electrons pass, in turn, to the next in a series of dynodes where their number is further multiplied. Depending on the number of the dynodes of the electron multiplier chain, their quality, and their applied potentials, a single electron passing through the defining aperture typically produces 10^6 to 10^7 electrons at the multiplier output anode. An example of a modern image dissector is shown schematically in Fig. 39.

To a first approximation, the resolution properties of an image dissector are determined solely by the geometric size and shape of the dissecting aperture. The predicted falloff in peak-to-peak signal modulation m for a dissector with various aperture sizes and shapes is shown as a bar pattern input in increasing spatial density R in line pairs per meter is shown in Fig. 40.

A lower limit on the effective aperture size of an image dissector occurs because of the finite emission energy of photoelectrons. The combination of a finite average lateral emission energy component V_{y,0} in volts with a finite average axial emission energy component V_{z,0} leads to an electron image on the dissector aperture plate (for a point optical input image) which is blurred according to an approximately Gaussian current density distribution called the “point-spread function” of the electron lens of the image dissector.

For a magnetically focused image dissector with a uniform electric accelerating field over a distance L_D in meters between the photocathode and an electron-transmissive mesh followed by an electric-field-free drift and deflection space (Fig. 39) of length L_D in meters prior to the aperture plate, the full width at half maximum W of the resulting point-spread function at the aperture plate in meters is given by

\[ W \approx 1.23L_D \left( \frac{V_{y,0}V_{z,0}}{V_{a}} \right)^{1/2} \left[ 1 + 0.42 \frac{L_D}{L_a} \right] \]

where V_{a} is the beam energy in volts within the drift and deflection space. To maintain this minimum (focused) beam size at the aperture requires a solenoidal magnetic field strength B in weber meter^{-2} given by

\[ B = \frac{\pi n (2m V_{a})^{1/2}}{I(t) e^{1/2}} \]

where n = number of beam loops between cathode and aperture plate= 1, 2, 3, ..., m = mass of the electron= 9.1 \times 10^{-31} \text{ kilogram, and } e = \text{electron charge} = 1.6 \times 10^{-19} \text{ coulomb.}

Given a dissector with the minimum useful aperture size (approximately one beamwidth W in diameter), the resulting maximum value of the spatial pattern density R_{max} in line pairs per meter which can be detected at a modulation ratio m \approx 0.05 is given by

\[ R_{\text{max}} \approx 0.7/W \]

**Signal:** For an input photocathode illuminance I_L in lumen meter^{-2}, the output signal current I_o from an image dissector is given by

\[ I_o = S \frac{G I_L}{t} \]

where S = photocathode luminous sensitivity in ampere lumen^{-1}, t = mesh transmission including collection efficiency losses between photocathode and first dynode ratio of the current transmitted by the mesh to the current incident on the mesh, and \( G \) = effective aperture area measured at the photo-
tential, and saturation charge results. This phenomenon accounts for the so-called "knee" in the signal-vs-illumination transfer curve (Fig. 42).

Because the target membrane is very thin of the order of microns, a charge distribution pattern on the image-section surface appears nearly simultaneously and identically on the scanning surface.

In the scanning section, an electron gun generates a highly aperture electron beam from a fraction to tens of micrometers in intensity. A solenoidal magnetic-focus coil and saddle-type deflection coils surrounding the scan section focus this beam on the imager target and move it across the target. Secondary electrons impinge on the target at very low velocity, giving rise to relatively few secondary electrons. The target acts somewhat as a retarding field electrode and reflects a large number of the beam electrons that have less than average axial velocity. These phenomena, small but finite secondary emission and reflection of slow beam electrons, limit scan-beam modulation to a maximum of about 30% at high light levels, and to 2 orders less at threshold. As will be shown later, the large unmodulated return beam current is the primary source of noise in the image section.

Another problem created by the retarding-field aspect of low-velocity target scanning appears when the deflected beam does not strike the target normally. Since the entire beam-velocity component normal to the surface is now reduced by the cosine of the angle of incidence, the effective beam impedance is greatly increased. To overcome this problem, the deflecting field between grids 4 and 5 is shaped such that the electron beam always approaches normal to the plane of the target at a low velocity. If the elemental area on the target is positive, then electrons from the scanning beam deposit until the charge is neutralized. If the elemental area is at cathode potential (corresponding to the beam), no electrons are deposited. In both cases the excess beam electrons are turned back and focused into a 5-stage electron multiplier. The charges existing on either side of the semiconductive target membrane will, by conductance, neutralize each other in less than one frame time.

Electrons turned back at the target form a return beam that has been amplitude-modulated in accordance with the charge pattern of the target.

The return beam is redirected by the deflection and focus fields toward the electron gun where it is returned and focused onto the target aperture for that gun, is a flat secondary-emitting surface comprising the first dynode of the electron multiplier. The return beam strikes this surface, generating secondary electrons in a ratio of approximately 4:1.

Grid 3 facilitates a more complete collection by dynode 2 of the secondary electrons emitted from dynode 1. The gain of the multiplier is high enough that in operation the limiting noise is the shot noise of the returned electron beam rather than the input noise of the video amplifier.

Signal and Noise: Typical signal output current for tube types 6880 and 6886 are shown in Fig. 42. The tube output is considerably so that the signal lights on the photocathode bring the signal output slightly over the knee of the signal-output curve. The spectral response of the types 6880 and 6886 is shown in Fig. 43. Note that when a Watten 6 filter is used with the tube, a spectral curve closely approximates that of the human eye is obtained.

From the standpoint of noise, the total television system can be represented as shown in Fig. 44, where $I_s$ is signal current, $I_n$ is total image-orthicon noise current, $I_{th}$ is thermal noise in $R_t$, $I_{ss}$ is shot noise in the input amplifier tube, $I_{in}$ is total input signal current, and $I_{rs}$ is shot-noise equivalent resistance of the input amplifier.

Fig. 42—Basic light-transfer characteristic for type 6880 and 6886 image orthicons. The curves are for small-area highlights illuminated by tungsten light, white fluorescent light, or daylight.
The signal current is an alternating-current signal superimposed on a direct beam current. This can be thought of as a modulation of the beam current. Properly adjusted tubes obtain as much as 30-per cent modulation.

$$I_m = m I$$

where \( m \) = multiplier gain and \( M \) = percentage modulation. If \( S/N \) is now rewritten

$$S/N = \left( \frac{m}{I_t} \right) \frac{4kT}{\pi^2} \left( \frac{\ln 2}{4\pi M} + \frac{R_1}{R_2} + \frac{R_2}{R_1} \right)^{1/2}$$

In typical television operation, the thermal noise of the load resistor and the shot noise of the first amplifier can be neglected.

**Focusing and Scanning Fields:** The electron optics of the scanning section of the tube are quite complicated and space does not permit the inclusion of the complete equations. A simple relationship between the strength of the magnetic focusing field and the magnetic deflection field is given below.

The image orthicon is usually operated with multiple-node focus in the scanning section. Working at a multiple-node focus not only demands more focus current but also more deflection current. Note the deflection path in Fig. 45. Let \( H = \text{horizontal} \) component of scanned area or target, \( L = \text{effective length of horizontal deflection field} \), \( H = \text{horizontal deflection field} \) (peak-to-peak value), and \( H = \text{focusing field} \). Then

$$H = H_1/l$$

for the image orthicon, \( H_1 = 1.25 \) inches, and \( L = 4 \) inches. Thus \( H = 75 \) gauss, and \( H = 25 \) gauss.

**Vidicons**

The vidicon is a small television camera tube that is used primarily for industrial television.

![Fig. 43—Spectral sensitivity of image orthicon.](image)

![Fig. 44—Deflection in image orthicon.](image)

![Fig. 45—Equivalent circuit for noise in orthicon and first amplifier stage.](image)

![Fig. 46—Vidicon construction.](image)

![Fig. 47—Input circuit for first-stage amplifier in vidicon circuit.](image)
A representative plot of amplitude response as a function of the number of television lines (per raster height) is shown in Fig. 48. The vidicon has somewhat more lag or image persistence than the image orthicon. This is the result of two factors. To obtain high-sensitivity surfaces, the photoconductive decay time is made as long as tolerable, since quantum efficiency is limited by the ratio of effective carrier lifetime to carrier transit time across the photoconductor. A second source of lag is simply the RC time constant of the target recharging circuit; that is, the target capacitance and the beam impedance.

The spectral response of most commercial vidicons, designated S-18, is more active than the human eye. Figure 49 compares these responses with the spectrum of a 2854°K tungsten source.

Fig. 48—Spectral response of vidicon.

Variations of the Vidicon

Interest in optical guidance and surveillance from air and spacecraft has given rise to a wide variety of vidicon camera tubes. To treat these variations in detail becomes encyclopedic, but the following gives some indication of the choices now available to the user.

Effective Sensitivity: True photoconductive tubes now offer sensitivities of 150-200 nanoamperes for 1-footcandle illumination with 20 nanoamperes dark current. Improved methods of deposition of photoconductors have made possible higher voltage operation without objectionable dark shading. Special devices using junction effects promise even better sensitivity.

Spectral Response: Available photoconductors, taken as a whole, provide sensitivity over the entire visible range with usual (7000) glass windows. Quartz window tubes offer useful sensitivity to below 2000 angstrom units. Numerous applications of direct excitation of photoconductors by X-radiation have been reported. High-voltage electron excitation (bombardment-induced conductivity) is also in use.

Size and Deflection: Vidicons are available in sizes ranging from 1/4 inch to 2 inches in diameter. Various combinations of deflection and focus are available.

Storage: A number of manufacturers have produced vidicons with long storage characteristics. Many are merely long-tube tubes; however, a few rely on high-resistivity materials or barrier layers to retain stored charge through minimal dark current. One such device, once exposed properly to a scene, regenerates the scene through readout over a period of the order of half an hour.

LIGHT-EMITTING TUBES

Cathode-Ray Tubes

A cathode-ray tube is a vacuum tube in which an electron beam, deflected by applied electric and/or magnetic fields, indicates by a trace on a fluorescent screen the instantaneous value of the retarding voltages and/or currents. A typical high-intensity cathode-ray tube with post-deflection acceleration is shown in Fig. 50.

Principle of Operation: The function of the cathode-ray tube is to convert an electrical signal into a visual display. The tube contains an electron gun structure (to provide a narrow beam of electrons) and a phosphor screen (refer to section on Phosphor Screens). The electron beam is directed to the phosphor screen and strikes it, causing light to be emitted in a small area or spot in proportion to the intensity of the electron beam. The beam intensity varies as a function of the electrical signal which is applied to the control element in the electron gun.

The electrical signal that controls the beam intensity corresponds to the desired picture information; therefore, in modulating the electron beam the individual picture elements can be reproduced on the phosphor screen in the same degree of black or white as in the original picture. Although one element is not enough to reproduce a picture, the same process is carried out for all picture elements in successive order, and each element is positioned correctly on the phosphor screen to reproduce the entire picture. Means are provided either internally or externally to the tube for positioning or deflecting the electron beam over the phosphor-screen area in some systematic fashion to reproduce the entire picture as a visual output.

Electric-Field Deflection: Deflection is proportional to the deflecting voltage, inversely proportional to the accelerating voltage, and in the direction of the applied field (Fig. 51). For structures using straight and parallel deflection plates, it is given by

\[ D = \frac{E_A L}{2E_0} A \]

where \( D \) = deflection in centimeters, \( E_A \) = accelerating voltage, \( E_0 \) = deflecting voltage, \( L \) = length of deflection plates or deflecting field in centimeters, \( L \) = length of deflecting field to screen in centimeters, and \( A \) = separation of plates.
Magnetic-Field Deflection: Deflection is proportional to the flux or the current in the coil, inversely proportional to the square root of the accelerating voltage, and at right angles to the direction of the applied field (Fig. 52). Deflection is given by

$$D = \frac{0.3LH}{(E_a)^{1/2}}$$

where $H = \text{flux density in gauss}$, and $L = \text{length of deflecting field in centimeters}$.

Deflection Sensitivity: The deflection sensitivity is linear up to the frequency where the phase of the deflecting voltage begins to reverse before an electron has reached the end of the deflecting field.

A well-designed shielded coil will require fewer amperes-turns.

Figure 53 is an example of good shield design, where

$$x = d_2/10.$$  

Storage Cathode-Ray Tubes

The storage cathode-ray tube produces a visual display of controllable duration. The tube has two electron guns, a phosphor viewing screen, and two fine-mesh metal screens. One of the electron guns is referred to as the writing gun and the other as the viewing gun. The web of one screen is coated with a thin dielectric material to form a surface on which the electron beam strikes information, and the other screen serves as an electron collector. A typical storage cathode-ray tube is shown in Fig. 54.

Principle of Operation: The writing gun emits a pencil-like electron beam which is intensity modulated by the information to be stored. The information is in the form of an electrical input signal. The storage surface is scanned by this high-resolution beam which actually strikes this surface. A positive-charge image, corresponding in value to the input signal pattern, is imposed on the storage surface where it remains until it decays or is erased. The storage screen forms an array of elemental electron guns, with each mesh hole considered as a control element of one of the guns. After the desired information has been stored on the storage mesh, the entire surface is flooded by an electron beam from the flooding gun. The value of positive charge deposited at each mesh aperture controls the amount of flooding beam current that can pass through the mesh aperture to the phosphor viewing screen. The current that passes through the mesh strikes the phosphor viewing screen, where a light output is observed in proportion to the bombarding-current density and the energy with which the electrons strike the phosphor. In other words, a grey scale is reproduced in the stored image. After the stored information has been observed or recorded, it is erased from the storage surface by flooding the storage surface with low-velocity electrons. Thus a net negative charge is deposited on each elemental area of the storage surface until flooding cathode potential is reached. The storage surface is then prepared for storing a new image.

Barrier-Grid Storage Tube

The barrier-grid storage tube is a form of cathode-ray tube in which information can be stored as an electrostatic charge. There is no visual display, since both the input and the output signals are electrical. The operation of this tube is closely similar to that of the storage cathode-ray tube. The tube (Fig. 55) consists of an electron gun for writing in and reading out the information, a curved target on which the information is stored, a collector electrode, and an associated electron optical system for focusing the signal electrons onto the collector electrode.

Principle of Operation: The target of the barrier-grid storage tube consists essentially of an array of elemental capacitors, which are charged or discharged by the action of the electron beam from the electron gun. During the write operation, an electrical signal (such as a video signal) is applied to a control electrode of the electron gun, which electrode in turn modulates the electron beam generated by the gun. This modulated beam is scanned across the target, and the information is stored on the target in the form of a pattern of charged areas. During the read cycle an unmodulated electron beam is scanned across the target. As this reading beam approaches each elemental charged area on the target, electrons leave the area and are attracted to the collector. This current constitutes the output signal.

Scan Converter

The scan converter is a tube in which information can be stored as an electrostatic charge. In the scan converter there is no visual display; its output as well as its input are electrical signals. The tube (Fig. 56) consists of two electron guns on opposite ends of the tube on the same axis, a

Fig. 52—Magnetic deflection.

Fig. 53—Magnetic focusing.

Fig. 54—Construction of storage cathode-ray tube.

Fig. 55—Construction of barrier-grid storage tube.

Fig. 56—Construction of scan converter.
collected mesh, and a storage screen comprised of a fine-mesh metal screen coated with a dielectric material serving as the storage surface. The holes of the screen remain open. Each electron gun directs a beam of electrons to the common storage mesh. Each beam is scanned over the storage surface by its own deflection system. One beam writes and the other reads, and it is possible for both modes of operation to occur simultaneously.

Principle of Operation: An electrical input signal, corresponding to the information to be stored, is applied to the writing gun. This signal controls the intensity of the electron beam which deposits or writes a pattern of electrostatic charge on the storage surface. This written information is read out in the form of an electrical output signal by the reading gun. The value of charge that has been deposited at each mesh aperture of the storage screen controls the amount of writing-beam current that can pass through the screen to the collector. The output signal thus varies as a function of the charge stored at each point on the storage screen. Either of the electron beams may be used to erase a stored picture after it has been read out thousands of times.

ELECTRON-TUBE CIRCUITS

APPLICATIONS OF ELECTRON TUBES

The advent of transistors is limiting the use of vacuum tubes to special applications. The voltage-handling capabilities of electron tubes satisfy the requirements for high-power oscillator, amplifier, and certain pulse service applications. Tubes are used in the high-power stages of radio and similar transmitters, as modulators of high-power radio-frequency amplifiers, and for specific conditions as pulse generators for radar and other pulse service equipment. Transistors and other semiconductor devices have largely replaced tubes in low-power applications.

GENERAL DESIGN

For quickly estimating the performance of a tube from catalog data, or for predicting the characteristics needed for a given application, the ratios given below may be used.

Table 1 gives correlating data for typical operation of tubes in the various amplifier classifications. From the table, knowing the maximum ratings of a tube, the maximum power output, currents, voltages, and corresponding load impedance may be estimated. Thus, taking for example a type P-124-A water-cooled transmitting tube as a class-C radio-frequency power amplifier and oscillator— the constant-current characteristics of which are shown in Fig. 1—published maximum ratings are as follows.

DC plate voltage:

\[ E_p = 20,000 \text{ volts} \]

DC grid voltage:

\[ E_g = 3000 \text{ volts} \]

DC plate current:

\[ I_p = 7 \text{ amperes} \]

RF grid current:

\[ I_g = 50 \text{ amperes} \]

Plate input:

\[ P_i = 135,000 \text{ watts} \]

Plate dissipation:

\[ P_d = 40,000 \text{ watts} \]

Maximum conditions may be estimated as follows. For \( n = 75 \text{ percent} \):

\[ P_i = 135,000 \text{ watts} \]

\[ E_p = 20,000 \text{ volts} \]

Power output \( P_e = n P_i = 100,000 \text{ watts} \).

Average dc plate current \( I_p = P_i/E_p = 0.7 \text{ amperes} \).
From tabulated typical ratio \( N_i / I = 4 \), instantaneous peak plate current \( I = 27 \) amperes.*

The rms value of the plate alternating-current component, taking ratio \( I_p / I = 1.2 \)

\[ I_p = 1.2I = 8 \text{ amperes.} \]

The rms value of the plate alternating-voltage component from the ratio \( E_p / E = 0.6 \) is \( E_p = 0.6E = 12,000 \text{ volts.} \)

The approximate operating load resistance \( R_l \) is now found from

\[ R_l = E_p / I_p = 1500 \text{ ohms.} \]

An estimate of the grid drive power required may be obtained by reference to the constant-current characteristics of the tube and determination of the peak instantaneous positive grid current \( N_i \), and the corresponding instantaneous total grid voltage \( E_i \). Taking the value of grid bias \( E_i \) for the given operating condition, the peak alternating grid voltage is

\[ E_i = E_p - E \]

from which the peak instantaneous grid drive voltage

\[ E_i = \text{peak grid voltage in volts.} \]

An approximation to the average grid drive power \( P_g \), necessarily rough due to neglect of negative grid current, is obtained from the typical ratio

\[ I_p / I = 0.2 \]

of dc to peak value of grid current, giving

\[ P_g = I_p E_p = 0.2I_p E_p \text{ watt.} \]

Plate dissipation \( P_p \) may be checked with published values since

\[ P_p = P_i - P_g \]

It should be borne in mind that combinations of published maximum ratings as well as each individual maximum rating must be observed. Thus, for example in this case, the maximum dc plate operating voltage of 20,000 volts does not permit operation at the maximum dc plate current of 7 amperes since this exceeds the maximum plate input rating of 135,000 watts.

Plate load resistance \( R_l \) may be connected directly in the tube plate circuit, as in the resistance-coupled amplifier, through impedance-matching elements as in audio-frequency transformer coupling, or effectively represented by a loaded parallel

\[ R_l = N R_i \]

in the case of a transformer-coupled stage, where \( N \) is the primary-to-secondary voltage transformation ratio. In a parallel-resonant circuit in which the output resistance \( R_i \) is connected directly in one of the reactance legs

\[ R_l = X / R_i = L / C \times Q \]

where \( X \) is the leg reactance at resonance (ohms), \( L \) and \( C \) are inductance in henries and capacitance in farads, respectively, and \( Q = X / R_i \).

**GRAPHIC DESIGN METHODS**

When accurate operating data are required, more precise methods must be used. Because of the nonlinear nature of tube characteristics, graphic methods usually are most convenient and rapid. Examples of such methods are given below.

**Class-C Radio-Frequency Amplifier or Oscillator**

Draw straight line from A to B (Fig. 1) corresponding to chosen dc operating point and grid voltages, and to desired peak alternating plate and grid voltage excursions. The projection of AB on the horizontal axis thus corresponds to \( E_p \). Using Chaffee's 11-point method of harmonic analysis, lay out on AB points to each of which correspond instantaneous plate currents \( i' \), \( i'' \), and \( i''' \) and instantaneous grid currents \( i' \), \( i'' \), and \( i''' \). The operating currents

<table>
<thead>
<tr>
<th>Function</th>
<th>Class A</th>
<th>Class B af (p-p)</th>
<th>Class B rf</th>
<th>Class C rf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate efficiency ( \eta ) (percent)</td>
<td>20-30</td>
<td>35-65</td>
<td>65-85</td>
<td>3.1</td>
</tr>
<tr>
<td>Peak instantaneous to dc plate current ratio ( N_i / I )</td>
<td>1.5-2</td>
<td>3.1</td>
<td>3.1-4.5</td>
<td>1.1</td>
</tr>
<tr>
<td>RMS alternating to dc plate current ratio ( I_p / I )</td>
<td>0.5-0.7</td>
<td>1.1</td>
<td>1.1-1.2</td>
<td>0.5-0.6</td>
</tr>
<tr>
<td>RMS alternating to dc plate voltage ratio ( E_p / E )</td>
<td>0.3-0.5</td>
<td>0.5-0.6</td>
<td>0.5-0.6</td>
<td>0.5-0.6</td>
</tr>
<tr>
<td>DC to peak instantaneous grid current ( I_p / E_i )</td>
<td>0.1-0.25</td>
<td>0.1-0.25</td>
<td>0.1-0.25</td>
<td>0.1-0.25</td>
</tr>
</tbody>
</table>
To illustrate the preceding exposition, a typical amplifier calculation is given below.

Operating requirements (carrier condition):

- \( E_d = 12000 \) volts
- \( P_v = 25000 \) watts
- \( \eta = 75 \) percent.

Preliminary calculation (refer to Tables 1 and 2):

\[ \frac{E_v}{E_p} = 0.8 \]

- \( \frac{E_p}{1.414} \times 12000 = 7200 \) volts
- \( E_p = 1.414 \times 7200 = 10000 \) volts
- \( I_v = P_v/E_p \)
- \( I_v = 25000/7200 = 3.48 \) amperes
- \( I_p = 3.48 \) amperes
- \( I_p/I_v = 1.2 \)
- \( I_p = 1.2 \times 3.48 = 4.17 \) amperes

Preliminary data:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Carrier</th>
<th>Crest</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_d ) (volts)</td>
<td>12000</td>
<td>12000</td>
</tr>
<tr>
<td>( E_p ) (volts)</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>( I_v ) (amp)</td>
<td>1.740</td>
<td>1.740</td>
</tr>
<tr>
<td>( I_p ) (amp)</td>
<td>5.1</td>
<td>10.2</td>
</tr>
<tr>
<td>( \eta ) (percent)</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>( R_f ) (ohms)</td>
<td>2060</td>
<td>1960</td>
</tr>
<tr>
<td>( R_e ) (ohms)</td>
<td>7100</td>
<td>7100</td>
</tr>
<tr>
<td>( \eta ) (percent)</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>

The effect of grid secondary emission is to lower the grid voltage at a rate of 1000 volts to locate point A.

For the following data are taken along AB:

- \( i'_C = 13 \) amp
- \( i''_C = 10 \) amp
- \( i''_C = 0.3 \) amp
- \( i''_C = 1.7 \) amp
- \( i''_C = -0.1 \) amp
- \( E_v = -1000 \) volts

Table 2—Class-C RF Amplifier Data for 100-Percent Plate Modulation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Carrier</th>
<th>Crest</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_d ) (volts)</td>
<td>12000</td>
<td>24000</td>
</tr>
<tr>
<td>( E_p ) (volts)</td>
<td>10000</td>
<td>20000</td>
</tr>
<tr>
<td>( I_v ) (amp)</td>
<td>1.740</td>
<td>1.740</td>
</tr>
<tr>
<td>( I_p ) (amp)</td>
<td>5.1</td>
<td>10.2</td>
</tr>
<tr>
<td>( \eta ) (percent)</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>( R_f ) (ohms)</td>
<td>2060</td>
<td>1960</td>
</tr>
<tr>
<td>( R_e ) (ohms)</td>
<td>7100</td>
<td>7100</td>
</tr>
<tr>
<td>( \eta ) (percent)</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>

The above procedure may also be applied to plate-modulated class-C amplifiers. The above data as applying to carrier conditions, the analysis is repeated for \( \text{amp} E_p = 2E_d \) and \( \text{amp} I_v = 2I_v \), keeping \( E_v \) constant. After a cut-and-try method has given a peak solution, it will be found that small grid bias as well as grid modulation are indicated to obtain linear operation.

\( R_e = (10000/5.1) = 1960 \) ohms

\( I_v = (1.7 + 1.7 (-0.1))/2 = 0.125 \) amp.

\( \eta = (1740 \times 0.255)/2 = 220 \) watts.

Operating data at 100-percent modulation is now calculated knowing that here

- \( E_v = 24000 \) volts
- \( R_e = 1960 \) ohms

For undistorted operation

- \( P_v = 4 \times 25500 = 102000 \) watts

The crest operating line \( \Delta B' \) is now located by trial so as to satisfy the above conditions, using the sample equations and method as for the carrier condition.

The effect of grid secondary emission is to lower the grid voltage at a rate of 1000 volts to locate point A.

For the following data are taken along AB:

- \( i'_C = 13 \) amp
- \( i''_C = 10 \) amp
- \( i''_C = 0.3 \) amp
- \( i''_C = 1.7 \) amp
- \( i''_C = -0.1 \) amp
- \( E_v = -1000 \) volts

The value of grid resistance required is given by

- \( R_e = (E_v - \text{amp} E_p)/(I_v - \text{amp} I_v) \)

The effect of grid secondary emission is to lower the grid voltage at a rate of 1000 volts to locate point A.

Class-B Radio-Frequency Amplifiers

A rapid approximate method is to determine by inspection from the tube characteristics \((i'_C - \alpha)\) the instantaneous current, \(i'_C\) and voltage \(e'_C\) corresponding to peak alternating voltage swing from operating voltage \(E_v\).

AC plate current:

\( \text{amp} I_v = i'_C/2 \)

DC plate current:

\( I_v = i'_C/\pi \)

AC plate voltage:

\( \text{amp} E_p = E_v - e'_C \)
Power output:
\[ P_v = \left( \frac{E_v - \epsilon_v}{\pi/4} \right) \]

Power input:
\[ P_i = E_i I_i/\pi \]

Plate efficiency:
\[ \eta = \left( \frac{\pi/4}{1 - (g_i/E_r)} \right) \]

Thus \( \eta \approx 0.6 \) for the usual crest value of \( E_r \approx 0.8 E_v \).

The same method of analysis used for the class-C amplifier may also be used in this case. The carrier and crest condition calculations, however, are now made from the same \( E_i \), the carrier condition corresponding to an alternating-voltage amplitude of \( E_r/2 \) such as to give the desired carrier power output.

For greater accuracy than the simple check of carrier and crest conditions, the radio-frequency plate currents \( I_{p'} \), \( I_{p''} \), \( I_{p'''}, I_{p'''} \), \( -I_{p'''} \), \( -I_{p'} \), and \( -I_{p''} \) may be calculated for seven corresponding selected points of the audio-frequency modulation envelope \( +M_{E_p}, +0.707 M_{E_p}, +0.5 M_{E_p}, 0, -0.5 M_{E_p}, -0.707 M_{E_p}, \) and \(-M_{E_p}\), where the negative signs denote values in the negative half of the modulation cycle. Designating
\[
S' = M_{I_p'}/(-M_{I_p'}) \quad D' = M_{I_p''}/(-M_{I_p''}) - 2M_{I_p'}^2
\]
the fundamental and harmonic components of the output audio-frequency current are obtained as
\[
M_{I_p'} = (S'/4) + [S'/2(2/2)I_p']^2 \quad (\text{fundamental})
\]
\[
M_{I_p''} = (S'' / 6) + (D''/4) + (D''/3)
\]
\[
M_{I_p'} = (S'' / 2) - (S''/3)
\]
\[
M_{I_p''} = (D''/8) - (D''/4) + (D''/3)
\]
\[
M_{I_p'} = (D''/12) - [S''/2(2/2)I_p'' + (S''/3)
\]
\[
M_{I_p''} = (D''/24) - (D''/4) + (D''/3).
\]

This detailed method of calculation of audio-frequency harmonic distortion may, of course, also be applied to calculation of the class-C modulated amplifier, as well as to the class-A modulated amplifier.

**Class-AB and B Audio-Frequency Amplifiers**

Approximate equations assuming linear tube characteristics (giving reference to Fig. 1, line CD) for a class-B audio-frequency amplifier

\[
M_{I_p} = \frac{i_i}{2}
\]

\[ P_o = \frac{M_{E_p} M_{I_p}}{2} \]

\[ P_i = (2/\pi) E_i M_{I_p} \]

\[ \eta = \frac{\pi/4}{(M_{E_p}/\epsilon_i)} \]

\[ R_{op} = 4(M_{E_p}/\epsilon_i) = 4R_i \]

Again an exact solution may be derived by use of the dynamic load line JKL on the \( (i_v - \epsilon_v) \) characteristic of Fig. 2. This line is calculated about the operating point \( K \) for the given \( E_i \) (in the same way as for the class-A case). However, since two tubes operate in phase opposition in this case, an identical dynamic load line MNO represents the other half cycle, laid out about the operating bias axisess point but in the opposite direction (see Fig. 2).

Algebraic addition of instantaneous current values of the two tubes at each value of \( e_i \) gives the composite dynamic characteristic OPL for the two tubes. Inasmuch as this curve is symmetrical about point \( P \), it may be analyzed for harmonics along a single half-curve PL, by the Morgenstern 8-point method. A straight line is drawn from \( P \) to \( L \) and ordinate plate-current differences \( a, b, c, d, f \) between this line and curve, corresponding to \( e_i, e''_i, e'''_i, e''''_i, e^{VI}_i \), and \( e^{VII}_i \) are measured. Ordinate distances measured upward from curve PL are taken positive.
Fundamental and harmonic current amplitudes and power are found from

\[
I_{f1} = I_{f1} + I_{f1} + I_{f1} + I_{f1},
\]

\[
I_{f2} = I_{f1} + I_{f1} + I_{f1} + I_{f1},
\]

\[
I_{f3} = I_{f1} + I_{f1} + I_{f1} + I_{f1},
\]

\[
I_{f4} = 0.4475 (b + f) - \frac{1}{3} I_{f1} + I_{f1} + I_{f1},
\]

\[
I_{f5} = 0.5 I_{f1} + I_{f1} + I_{f1} + I_{f1},
\]

\[
I_{f6} = 0.577 \cdot I_{f1} + I_{f1} + I_{f1} + I_{f1},
\]

Even harmonics are not present due to dynamic characteristic symmetry. The direct-current and power-input values are found by the 7-point analysis from curve PL and doubled for two tubes.

**CLASSIFICATION OF AMPLIFIER CIRCUITS**

The classification of amplifiers in classes A, B, and C is based on the operating conditions of the tube. Another classification can be used, based on the type of circuits associated with the tube. A tube can be considered as a four-terminal network with two input terminals and two output terminals. One of the input terminals and one of the output terminals are usually common; this common junction or pins is usually called "ground."

When the common point is connected to the filament or cathode of the tube, we speak of a grounded-grid circuit, the most conventional type of vacuum-tube circuit. When the common point is the grid, we speak of a grounded-cathode circuit, and when the common point is the plate or anode, we can speak of the grounded-anode circuit.

This last type of circuit is most commonly known by the name of cathode-follower.

A fourth and most general class of circuit is obtained when the common point or ground is not directly connected to any of the three electrodes of the tube. This condition encountered at

\[
Z_c = \text{load impedance to which output terminals of amplifier are connected}
\]

\[
E_i = \text{phaser input voltage to amplifier}
\]

\[
E_o = \text{output voltage across load impedance}
\]

\[
A = \frac{E_o}{E_i}
\]

\[
Y_{i} = \text{input admittance to input terminals of amplifier}
\]

\[
\omega = 2\pi f \left(\text{frequency of excitation voltage} E_i\right)
\]

\[
j\omega = \left(-1\right)^{j\omega}
\]

**AMPLIFIER PAIRS**

The basic amplifier classes are often used in pairs or combinations for special characteristics. The availability of dual triodes makes these combined forms especially useful.

**Grounded-Cathode—Grounded-Plate**

This pairing provides the gain and 180° phase reversal of a grounded-cathode stage (Fig. 3) with a low source impedance at the output terminals. It is especially useful in feedback circuits or for amplifiers driving a low or unknown load impedance. In tuned amplifiers, the possibility of oscillation must be considered. Direct coupling is useful for pulse work, permitting large positive input and negative feedback excitations.

**Grounded-Cathode—Grounded-Grid (Cathode-Coupled)**

Direct coupling is usual, making a very simple structure. Several modified forms are possible with special characteristics.

**Cathode-Coupled Amplifier:** As a simple amplifier, Rf and input $E_i$ (Fig. 4) are short-circuited. Output $E_o$ is in phase with input $E_i$. Gain (with $R_o > 1/m_i$) is given by $A = m_i R_f/2$. Even harmonic distortion is reduced by symmetry, as in a push-pull stage. Due to the in-phase input and output relations, this circuit forms the basis for various R-C oscillators and the class of cathode-coupled multivibrators.

**Symmetrical Clipper:** With suitable bias adjustment, symmetrical clipping or limiting occurs between $V_i$ cutoff and $V_o$ cutoff, without drawing grid current.

**Differential Amplifier:** With input supplied to $E_1$ and $E_2$, the output $E_o$ responds (approximately) to the difference $E_1 - E_2$. Balance is improved by constant-current supply to the cathode such as a high value of $R_i$ (preferably connected from a highly negative supply) or a constant-current pentode. The signal to $E_i$ should be slightly attenuated for precise adjustment of balance.

**Phase Inverter:** With $R_2$ and $R_2$ both used, approximately balanced (push-pull) outputs ($E_1$ and $E_2$) are obtained from either input $E_i$ or $E_i$. As a phase inverter (paraphase), one input ($E_i$) is used, the other being grounded, and $R_2$ is made slightly less than $R_2$ to provide exact balance.

**Grounded-Cathode—Grounded-Grid (Cascade)**

This circuit (Fig. 5) has characteristics somewhat resembling the pentode, with the advantage that no screen current is required. $V_3$ serves to isolate $V_4$ from the output load $R_o$, giving voltage gain equation

\[
A = \frac{\mu R_f}{(r_{ps} + R_f)/(\mu + 1)}
\]

For $R_f < \mu r_{ps}$,

\[
A = \mu
\]

For $R_f > \mu r_{ps}$,

\[
A = \mu r_{ps}
\]

As an rf amplifier, the grounded-grid stage $V_3$ drastically reduces capacitive feedback from output to input, without introducing partition noise (as produced by the screen current of a pentode). Shot noise contributed by $V_3$ is negligible due to the highly degenerative effect of $r_{ps}$ in series with the cathode. The noise figure is thus approaches the theoretical noise of $V_3$ used as a triode, without the undesirable effects of triode plate-grid capacitance.

Because of the 180° phase relation of input and output, this circuit is also valuable in audio feedback circuits, replacing a single stage with considerable increase in gain (for high values of $R_f$).

The grid of $V_4$ provides a second input connection $E_i'$ useful for feedback or for gating. The voltage gain from $E_i'$ to the output is considerably reduced, being given by

\[
A = R_f/(R_4 + \mu r_{ps})
\]

For $R_f < \mu r_{ps}$,

\[
A \approx R_f/r_{ps}
\]

For $R_f > \mu r_{ps}$,

\[
A \approx \mu
\]

**CATHODE-FOLLOWER DATA**

**General Characteristics**

(A) High-impedance input, low-impedance output.

(B) Input and output have one side grounded.

(C) Good wide-band frequency and phase response.

(D) Output is in phase with input.

(E) Voltage gain or transfer is always less than one.

(F) A power gain can be obtained.

(G) Input capacitance is reduced.
sion for voltage gain of a cathode-follower (including $C_{a}$) is given by

$$A = \frac{\mu Z_a + Z_{g} + Z_{0}}{\mu Z_{a} + Z_{0}} \left[ 1 + \frac{Z_{0} + Z_{g}R_{s}}{Z_{a}R_{s}} \right]$$

The input admittance (Table 3)

$$Y_{i} = j\omega C_{a} + \left( 1 - A \right) C_{0}$$

may contain negative-resistance terms causing oscillation at the frequency where an inductive grid circuit resonates the capacitive $Y_{i}$ component.

The use of a simple triode (or pentode) grounded-cathode circuit with a load resistor equal to $Z_{0}$ provides an equally good match with slightly higher gain ($g_{m}R_{s}$), but will overload at a lower maximum voltage. The anode-follower provides output approximating the cathode-follower without the risk of oscillation.

### General Case (Fig. 6)

Transfer $= R_{out}/E_{in} = g_{m}R_{s} \left[ \frac{Z_{0} + R_{s}}{1 + \left( R_{1}/r_{p} \right)} \right]$

- $R_{out} =$ output resistance
- $r_{p} = \left( \mu + 1 \right)$ or approximately $1/g_{m}$
- $g_{m} =$ transconductance in mhos
  (1000 microhms $= 0.001$ mho)
- $R_{1} =$ total load resistance
- Input capacitance $= C_{a} + \left[ C_{0}/(1 + g_{m}R_{s}) \right]$

### Specific Cases

(A) To match the characteristic impedance of the transmission line, $R_{out}$ must equal $Z_{0}$ (Fig. 7).
(B) If $R_{out}$ is less than $Z_{0}$, add resistor $R_{4}$ in series (Fig. 8) so that $R_{4} = Z_{0} - R_{out}$.
(C) If $R_{out}$ is greater than $Z_{0}$, add resistor $R_{4}$ in parallel (Fig. 9) so that $R_{4} = Z_{0}R_{out}/(Z_{0} - R_{out})$.

Note 1: Normal operating bias must be provided.

To couple a high impedance into a low-impedance transmission line, for maximum transfer choose a tube with a high $g_{m}$.

Note 2: Oscillation may occur in a cathode-follower if the source becomes inductive and load capacitive at high frequencies. The general expression for voltage gain of a cathode-follower (including $C_{a}$) is given by

$$A = \frac{\mu Z_a + Z_{g} + Z_{0}}{\mu Z_{a} + Z_{0}} \left[ 1 + \frac{Z_{0} + Z_{g}R_{s}}{Z_{a}R_{s}} \right]$$

The input admittance (Table 3)

$$Y_{i} = j\omega C_{a} + \left( 1 - A \right) C_{0}$$

may contain negative-resistance terms causing oscillation at the frequency where an inductive grid circuit resonates the capacitive $Y_{i}$ component.

The use of a simple triode (or pentode) grounded-cathode circuit with a load resistor equal to $Z_{0}$ provides an equally good match with slightly higher gain ($g_{m}R_{s}$), but will overload at a lower maximum voltage. The anode-follower provides output approximating the cathode-follower without the risk of oscillation.

### NEGATIVE FEEDBACK

The following quantities are functions of frequency with respect to magnitude and phase.

- $E$, $N$, $D =$ signal, noise, and distortion output voltage with feedback
- $e$, $n$, $d =$ signal, noise, and distortion output voltage without feedback
- $A =$ voltage amplification magnitude of amplifier at a given frequency
- $A =$ amplification including phase angle (complex quantity)
- $3 =$ fraction of output voltage fed back (complex quantity) for usual negative feedback; $3$ is negative
- $\phi =$ phase shift of amplifier and feedback circuit at a given frequency.

Fig. 10—In negative-feedback amplifier considerations $3$, expressed as a percentage, has a negative value. A line across the $\beta$ and $A$ scales intersects the center scale to indicate change in gain. It also indicates the amount, in decibels, the input must be increased to maintain original output.

| PERCENT FEEDBACK | 80 |
| CHANGE IN GAIN | 0.001 |
| ADDITIONAL GAIN NEEDED TO MAINTAIN ORIGINAL GAIN (DECIBELS) | 0.001 |
| ORIGINAL AMPLIFIER GAIN | 80 |
| ORIGINAL AMPLIFIER GAIN (DECIBELS) | 0.001 |
In the general case when \( \phi \) is not restricted to 0 or \( \pi \)
\[
\text{voltage gain} = \frac{A_i}{(1 + |A_j| + |A_j| \cos \phi)^{1/2}}
\]
\[
\text{change of gain} = \frac{1}{(1 + |A_j| + |A_j| \cos \phi)^{1/2}}
\]
Hence if \( |A_j| \gg 1 \), the expression is substantially independent of \( \phi \).

On the polar diagram relating \( |A_j| \) and \( \phi \) (Nyquist diagrams), the system is unstable if the point \((1, 0)\) is enclosed by the curve. Examples of Nyquist diagrams for feedback amplifiers will be found in the chapter on Feedback Control Systems.

**DISTORTION**

A rapid indication of the harmonic content of an alternating source is given by the distortion factor, which is expressed as a percentage.

(Distortion factor)
\[
= \left[ \frac{\text{sum of squares of amplitudes of harmonics}}{\text{square of amplitude of fundamental}} \right]^{1/2} \times 100 \text{ percent}
\]

If this factor is reasonably small, say less than 10 percent, the error involved in measuring it is
\[
= \left[ \frac{\text{sum of squares of amplitudes of harmonics}}{\text{sum of squares of amplitudes of fundamental and harmonics}} \right]^{1/2} \times 100 \text{ percent}
\]

is also small. This latter is measured by the distortion-factor meter.

**RELAXATION OSCILLATORS**

Relaxation oscillators are a class of oscillator characterized by a large excess of positive feedback, causing the circuit to operate in abrupt transitions between two blocked or overloaded end-states. These end-states may be stable, the circuit remaining in such condition until externally disturbed; or quasi-stable, recovering (after a period determined by coupling-circuit time constants and bias) and switching back to the opposite state. Relaxation oscillators are classified as bistable, monostable, or astable according to the number of stable end-states. Most circuits are adaptable to all three forms. Multistate devices are also possible. A wide variety of circuit arrangements is possible, including multivibrators, blocking oscillators, trigger circuits, counters, and circuits of the phasorotron, santron, and sanphon class. Relaxation oscillators are often used for counting and frequency division, and to generate nonsinusoidal waveforms for timing, triggering, and similar applications.

**Multivibrators**

A number of multivibrator circuits are formed from three basic two-stage amplifiers (grounded-cathode-grounded-cathode, grounded-plate-grounded-grid, and grounded-cathode-grounded-grid, or combinations of these types), that readily provide the needed positive feedback with simple resistance or resistance-capacitance coupling. End-states may be any two of the four "blocked" conditions corresponding to cutoff or saturation in either stage. In general, the duration of a quasi-stable state will be determined by the exponential decay of charge stored in a coupling-circuit time constant, (the circuit switching back to the opposite state when the saturated or the cutoff tube recovers gain), while stable states are produced by direct coupling with bias sufficient to hold one tube inoperative. The memory effect of charge storage also operates in the case of stable end-states to ensure completion of transfer across the unstable region. The timing accuracy of an astable or quasi-stable multivibrator is considerably improved by supplying the grid resistors from a high positive voltage (B+). The recovery from a cutoff condition thereby becomes an exponential towards a voltage much higher than the operating point, terminating in switchover when the output tube conducts. Grid conduction serves to clamp the capacitor voltage during the conducting state, erasing residual charge from the previous state. The starting condition for the next transition is thus more precisely determined and the linearity of the exponential recovery is improved by the essentially constant-current discharge (since the range from cutoff to zero bias represents a smaller fraction of total charge). The grid-circuit time constant must be appropriately increased to obtain the same dwell time.

**Bistable Circuits**

Bistable circuits are especially suited for binary counters and frequency dividers and as trigger circuits to produce a step or pulse when an input signal passes above or below a selected amplitude.

**Symmetrical Bistable Multivibrator**

The circuit is shown in Fig. 11. Trigger signal may be applied to both plates, both grids, or to both plate and grid inputs.

**Binary Counter Stage**

An adaptation of the symmetrical bistable multivibrator is shown in Fig. 12. Alternative trigger inputs are shown with corresponding outputs to drive a following stage. The use of coupling diodes \( V_i \) reduces the tendency of \( C_i \), \( C_j \) in the circuit of Fig. 11 to cause misleadingly unbalanced stored charge. Tubes \( V_i \) and \( V_j \) illustrate the application of clamping diodes, especially useful in high-speed circuits, to fix critical operating voltages. Pentodes with plate and grid clamping are suitable for very high speeds.

**Schmitt Trigger**

The circuit of Fig. 13 has the property that an output of constant peak value (a flat-topped pulse) is obtained for the period that the input waveform exceeds a specific voltage.

**Monostable Circuits**

Monostable multivibrators are useful for driven-sweep, pulse, and timing-wave generators. The absence of time constants and residual charge "memory" in the stable state reduces jitter when they are driven with irregularly spaced timing signals. Monostable versions may be derived from...
all of the foregoing bistable multivibrators by elimination of the direct (dc) coupling to one or the other grid. The circuit of Fig. 14 with R omitted is commonly used for pulse generation.

Most astable circuits can be made monostable by sufficient inequality of bias. The circuit of Fig. 17 is an example.

Sweep waveforms can be produced by integration of pulse outputs. The phantastron class of Miller sweep generators is also particularly useful for this purpose.

**Driven (One-Shot) Multivibrator:** The circuit is given in Fig. 15. Equations are

\[ f_{wo} = f_s \]

\[ f_{wo} = \text{multivibrator frequency in hertz} \]

\[ f_s = \text{synchronizing frequency in hertz} \]

Conditions of operation are

\[ f_s > f_r \text{ or } \tau_s < \tau_r \]

where

\[ f_s = \text{free-running frequency in hertz} \]

\[ \tau_s = \text{synchronizing period in seconds} \]

\[ \tau_r = \text{free-running period in seconds} \]

At the control resistor \( R_4 \)

\[ \tau_m = \beta_4 \log[(E_m - E_{m1} + E_s)/E_{m2} + E_s] \]

where

\[ E_m = \text{plate-supply voltage} \]

\[ E_{m1} = \text{minimum ac voltage on plate of V_1} \]

\[ E_s = \text{bias voltage of V_2} \]

\[ E_s = \text{cutoff voltage corresponding to E_m} \]

**Regenerative Clipper:** Bias on the first grid places the circuit of Fig. 14 in the center of the unstable region, giving regenerative clipping.

**Phantastron:** The phantastron circuit is a form of monostable multivibrator with similarities to the Miller sweep circuit. It is useful for generating very short pulses and linear sweeps. It uses a characteristic of pentodes: that while cathode current is determined mainly by control-grid potential, the screen-grid, suppressor-grid, and plate potentials determine the division of current between plate and screen. In certain tubes, such as the 6AS6, the transconductance from suppressor grid to plate is sufficiently high so that the plate current may be cut off completely with a small negative bias on the suppressor.

A typical phantastron circuit is shown in Fig. 16. During operation it switches between two states of interest.

(A) **Stable:** The control grid is slightly positive and draws current. Cathode current is maximum and the suppressor is biased negatively to plate-current cutoff by the cathode current in \( R_6 \). The plate is at a high potential determined by the clamping diode, and the screen potential is low.

(B) **Unstable:** When a positive trigger is applied to the suppressor grid (or a negative trigger to the control grid, cathode, or plate) the plate conducts, driving the control grid negative, reducing the cathode current, and taking most of the screen current. The plate potential then runs down linearly as in the Miller circuit.

The end of this period comes when the control grid goes positive again, resulting in increase of cathode current, suppressor cutoff, and heavy screen current.

In the circuit shown, the pulse width is adjustable from 0.3 to 0.6 microsecond. For longer pulses, it is possible to get a wide range of control both by varying \( R_1 \) and \( C \) and by varying the plate-clamping potential.

Decreasing \( R_6 \) results in astable operation.

**Astable Circuits**

The operating principles of the multivibrator and the exponential recovery from quiescent states are illustrated by the analysis of the free-running multivibrator.

**Free-Running Zero-Bias Symmetrical Multivibrator:** Exact equation for semiperiod (Figs. 17 and 18)

\[ \tau = [R_6 + (R_1R_2)/(R_2 + R_3)] C_1 \times \log[(E_s - E_m)/E_s] \]

where

\[ \tau = \text{period in seconds} \]

\[ \tau_1 = \text{semiperiod in seconds} \]

\[ R_0 = \text{plate resistance of tube in ohms} \]

\[ E_s = \text{plate-supply voltage} \]

\[ E_m = \text{minimum alternating voltage on plate} \]

\[ E_s = \text{cutoff voltage corresponding to E_s} \]

\[ C = \text{capacitance in farads} \]
Approximate equation for semiperiod, where
\[ R_{p} \gg \left[ R_{f} + \frac{1}{\tau_{f}} \right] \]
\[ \tau_{f} = R_{p} C_{1} \log_{2} \left( \frac{E_{c} - E_{m}}{E_{c}} \right). \]

Equation for buildup time is
\[ \tau_{b} = 4 \left( R_{f} + R_{p} \right) C \]
\[ = 95 \text{ percent of peak value.} \]

**Free-Running Zero-Bias Unsymmetrical Multivibrator:** See asymmetrical multivibrator (above) for circuit and terminology; the waveforms are given in Fig. 18. Equations for fractional periods are
\[ \tau_{f} = \left[ R_{f} + \left( R_{f} + R_{p} \right) \right] C \times \log_{2} \left( \frac{E_{c} - E_{m}}{E_{c}} \right) \]
\[ \tau_{s} = \left[ R_{f} + \left( R_{f} + R_{p} \right) \right] C \times \log_{2} \left( \frac{E_{c} - E_{m}}{E_{c}} \right) \]
\[ = \tau_{f} + \tau_{s} = 1/f. \]

**Free-Running Positive-Bias Multivibrator:** Equations for fractional period (Fig. 20) are
\[ \tau_{f} = \left[ R_{f} + \left( R_{f} + R_{p} \right) \right] C \times \log_{2} \left( \frac{E_{c} - E_{m} + E_{a}}{E_{c} + E_{a}} \right) \]
\[ \tau_{s} = \left[ R_{f} + \left( R_{f} + R_{p} \right) \right] C \times \log_{2} \left( \frac{E_{c} - E_{m} + E_{a}}{E_{c} + E_{a}} \right) \]

where
\[ f = \text{ repetition frequency in hertz} \]
\[ E_{s} = \text{ maximum negative grid voltage} \]
\[ E_{g} = \text{ positive bias voltage} \]
\[ \alpha = \text{ grid time constant in seconds} \]
\[ \epsilon = 2.718 = \text{ base of natural logs} \]
\[ \theta = \text{ decrement of wave} \]

(A) Use strong feedback
\[ = E_{s} \text{ is high.} \]
(B) Use large grid time constant
\[ = \alpha \text{ is large.} \]
(C) Use high decrement (high losses)
\[ = \theta \text{ is high.} \]

Pulse width is
\[ \tau = 2 \left( \frac{L}{C} \right)^{1/2} \]

**Astable Positive-Bias Wide-Frequency-Range Blocking Oscillator:** Typical circuit values (Fig. 24) are
\[ L = 0.5 \text{ to } 5 \text{ megohms} \]
\[ C = 50 \text{ picofarads to } 0.1 \text{ microfarad} \]
\[ R_{s} = 10 \text{ to } 200 \text{ ohms} \]
\[ R_{s} = 50 \text{ k ohms to } 250 \text{ k ohms} \]
\[ \Delta f = 100 \text{ hertz to } 100 \text{ kilohertz.} \]

**Monostable Blocking Oscillator:** Operating conditions (Fig. 25) are:
(A) Tube off unless positive voltage is applied to grid.
(B) Signal input controls repetition frequency.
(C) \( E_{s} \) is a high negative bias.
Synchronized Astable Blocking Oscillator: Operating conditions (Fig. 26) are
\[ f_s < f_s \] or \[ \tau_s > \tau_s \]
where
- \( f_s \) = free-running frequency in hertz
- \( f_s \) = synchronizing frequency in hertz
- \( \tau_s \) = free-running period in seconds
- \( \tau_s \) = synchronizing period in seconds.

Gas-Tube Oscillators
A simple relaxation oscillator is based on the negative-resistance characteristic of a glow discharge, the two end-states corresponding to ignition and extinction potential of the discharge. Two astable forms are discussed. The circuit of Fig. 27 may also be used with a simple diode (neon lamp), omitting the grid resistor and bias. The circuit of Fig. 28 may be made monostable if the supply voltage is less than the ignition voltage at the selected bias.

Astable Gas-Tube Oscillator: This circuit is often used as a simple generator of the sawtooth waveform necessary for the horizontal deflection of a cathode-ray-oscilloscope beam. Equation for period (Fig. 27)
\[ \tau = \alpha RC (1 + \alpha/2) \]

where
- \( \tau \) = period in hertz
- \( \alpha = (E_i - E_o)/(E - E_s) \)
- \( E_i \) = ignition voltage
- \( E_o \) = extinction voltage
- \( E \) = plate-supply voltage.

Velocity error
\[ = \text{change in velocity of cathode-ray-tube spot/\ trace period} \]
Maximum percentage error = \( \alpha \times 100 \)
if \( \alpha \ll 1 \).
Position error
\[ = \text{deviation of cathode-ray-tube trace from linearity.} \]
Maximum percentage error = \( (\alpha/8) \times 100 \)
if \( \alpha \ll 1 \).

Synchronized Astable Gas-Tube Oscillator: Conditions for synchronization (Fig. 28) are
\[ f_s = N \cdot f_n \]
where
- \( f_s \) = free-running frequency in hertz
- \( f_s \) = synchronizing frequency in hertz
- \( N \) = an integer.

For \( f_s \neq N \cdot f_n \), the maximum \( \delta f \) before slipping is given by
\[ \left( \frac{E_o}{E_s} \right) \left( \frac{\delta f_s}{f_s} \right) = 1 \]
where
- \( \delta f_s = f_s - f_s \)
- \( E_o \) = free-running ignition voltage
- \( E_s \) = synchronizing voltage referred to plate circuit.